DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2022

Subject: Physics

Paper: PHY/CC/TH/201

Quantum Mechanics II

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks.
Answer any two questions from each group
(Use separate answer scripts for each group)

Group – A

(Answer any two)

1. (a) Dirac Hamiltonian is given by, $H = \vec{\alpha} \cdot \vec{p} + \beta m$, show that α_i and β are hermitian, traceless matrices of even dimensionality with eigenvalues ± 1. [Symbols carry their usual meanings.]

(b) Consider $\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$ as solution to Dirac equation in two-component formalism under non-relativistic approximation $(v \ll c)$ with ϕ and χ as the "large" and "small" components of the electron wave function, respectively.

Show that the Dirac equation for an electron in an electromagnetic field, $A^{\mu} = (V,\ \vec{A})$ reduces to the Schrodinger-Pauli equation,

$$\left(\frac{1}{2m}\left(\vec{p} - \frac{e}{c}\,\vec{A}\right)^2 - \frac{e\hbar}{2mc}(\vec{\sigma} \cdot \vec{B}) + eV\right)\phi = i\hbar\frac{\partial \phi}{\partial t}$$

[Symbols carry their usual meanings]. Mention the assumptions and approximations clearly. 5 + 5 = 10

2. (a) Construct the rotation matrix for spin ½ system. The rotation operator about an arbitrary axis is represented by $U^{(j)}(\theta,\ \hat{n}) = e^{-\frac{i}{\hbar}(\vec{J}\cdot\hat{n})\theta}$ where, $\hat{n}$ denotes the arbitrary direction of the axis and θ stands for the angle of rotation about $\hat{n}$.

(b) Consider a sequence of Euler rotations represented by:

$$U^{\left(\frac{1}{2}\right)}(\alpha, \beta, \gamma) = e^{-\frac{i}{2}\sigma_3\alpha}\, e^{-\frac{i}{2}\sigma_2\beta}\, e^{-\frac{i}{2}\sigma_3\gamma}$$

$$= \begin{pmatrix} e^{-\frac{i}{2}(\alpha+\gamma)} \cos\frac{\beta}{2} & -e^{-\frac{i}{2}(\alpha-\gamma)} \sin\frac{\beta}{2} \\ e^{\frac{i}{2}(\alpha-\gamma)} \sin\frac{\beta}{2} & e^{\frac{i}{2}(\alpha+\gamma)} \cos\frac{\beta}{2} \end{pmatrix}$$

Assuming the sequence of rotations to be equivalent to a single rotation about some arbitrary axis denoted by $\hat{n}$ through an angle θ, find θ.

(c) The expectation value of a component of a vector operator is given by:

$$\langle j', m \,| T(1,0) | j, m\rangle = A.$$

Find the expectation values of $\langle j', m' | T(1,+1) | j, m\rangle$ and $\langle j', m' | T(1,-1) | j, m\rangle$ in terms of appropriate CG coefficients and A for $j = j' = 1$.

4 + 3 + 3 = 10

3. (a) Obtain the conditions for Dirac equation to remain invariant under parity transformation. How does the positive energy four-spinor solution of Dirac equation transform under this transformation?

(b) (c) Using the recursion relations find the CG coefficients for addition of angular momentum for $j_1 = \frac{1}{2}$ and $j_2 = \frac{1}{2}$.

(c) Considering infinitesimal continuous transformation implemented on a quantum system, show that the generators of the corresponding transformation must commute with the Hamiltonian of the corresponding quantum system.

5 +3 +2 = 10

Group – B

(Answer any two)

4. (a) Why does the electric dipole approximation for interaction of electromagnetic radiation valid for radiation in visible region? Mention one drawback of TDPT.

(b) What is first Born approximation? Starting from standard expression of scattering amplitude $f(\theta,\phi)$ using 1st Born approximation, obtain the expression of differential scattering cross section for Rutherford scattering. (3+1) + (2+4) = 10

5. (a) Write down all possible wave functions (spin-orbitals) of the 1st excited state of He atom.

(b) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, for a constant perturbation: $H'(t') = H'$ for $0 \leq t' \leq t$; zero otherwise, obtain an expression for the transition probability. Sketch the variation of probability with energy/frequency. Obtain the expression of probability when $t \rightarrow \infty$. 4 + (3+1+2) = 10

6. (a) Using Fermi golden rule obtain an expression for differential scattering cross section of a particle by spherically symmetric potential.

(b) Find the total scattering cross section of a particle, in the low energy limit, by a square well potential of the form

$$V(r) = -V_0, \quad \text{for } 0 < r < a$$
$$= 0, \quad \text{for } r \geq a, \ a = \text{width of the potential well.}$$

What is Ramsour-Towncend effect? 4 + (5+1) = 10

Formulae:

1. Recursion relation among CG coefficients:

$$\sqrt{(j \mp m)(j \pm m + 1)}\langle j_1 j_2; m_1 m_2 | j_1 j_2; j, m \pm 1\rangle$$
$$= \sqrt{(j_1 \pm m_1)(j_1 \mp m_1 + 1)}\langle j_1 j_2; m_1 \mp 1, m_2 | j_1 j_2; jm\rangle$$
$$+ \sqrt{(j_2 \pm m_2)(j_2 \mp m_2 + 1)}\langle j_1 j_2; m_1, m_2 \mp 1 | j_1 j_2; jm\rangle$$

2. Wigner-Eckart theorem:

$$\langle \alpha', j', m' | T(k,q) | \alpha, j, m\rangle = \langle j, k; m, q | j, k; j', m'\rangle \langle \alpha', j' || T(k) || \alpha, j\rangle$$

3. C.G. coefficient tables:

1×1

m1	m2			
		2		
		+2		
+1	+1	1		
		2	1	
		+1	+1	
+1	0	1/2	1/2	
0	+1	1/2	-1/2	
		2	1	0
		0	0	0
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3
		2	1	
		-1	-1	
0	-1	1/2	1/2	
-1	0	1/2	-1/2	
		2		
		-2		
-1	-1	1		

j1 j2 J M

1×1/2

m1	m2		
		3/2	
		+3/2	
+1	+1/2	1	
		3/2	1/2
		+1/2	+1/2
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3
		3/2	1/2
		-1/2	-1/2
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3
		3/2	
		-3/2	
-1	-1/2	1	

m1 m2

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M.Sc. (1st Year) 2nd Semester Examination, 2022

Subject: Physics

Paper: Statistical Mechanics

PHY/CC/TH/202

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

Use separate answer scripts for each group.

Answers to the questions should be written in the candidates own words as far as practicable.

Group – A

Answer any two questions.

1. (a) What is Gibb's paradox? Why does it arrive and how is it avoided?

   (b) Derive Sackur-Tetrode equation for entropy of an ideal gas.

   (c) Show that the partition function Z_N for an extreme relativistic gas consisting of $N-$ non-interacting molecules with energy-momentum relationship is given by $\epsilon = cp$

$$Z_N = \frac{1}{N!}\left[8\pi V\left(\frac{kT}{hc}\right)^3\right]^N.$$

Hence deduce pressure of the gas.

[(1+1+1)+3+(3+1)=10]

2. (a) Consider a system with two Bosons particles that can occupy three energy levels: -2ϵ (non-degenerate), ϵ (non-degenerate) and 2ϵ (doubly degenerate). Find

   i. Partition function of the system

   ii. Probability that the system has energy 2ϵ

   iii. Free energy of the system.

   (b) Establish the following relations

   i. $S = k\frac{\partial}{\partial T}(T\ln Z)$

ii. $N = kT\frac{\partial}{\partial\mu}(\ln Z)$
where Z is grand canonical partition function of the system and other terms have their usual meanings.

[(3+2+1)+(2+2)=10]

3. (a) State and prove Liouville's theorem. Discuss its physical significance.
   (b) Show that for a system in a canonical ensemble the mean square fluctuation in energy can be written as $< (\Delta E)^2 >= k_B T^2 C_V$ where C_V is specific heat at constant volume.

[(1+4+1)+4=10]

Group – B

Answer any two questions.

4. Consider a closed 1-D Ising lattice of N spins (σ) with periodic boundary condition ($\sigma_{N+1} = \sigma_1$) in presence of an external magnetic field (of strength B). The interaction energy strength between the two consecutive spins is denoted as I and the magnetic moment for the spins is expressed as μ. It is given that each of the spins can only take two values viz. $\sigma = \pm 1$ (spin up or spin down) and the system is in thermal equilibrium at temperature T.
   (a) Write down the Hamiltonian of the system.
   (b) Obtain the partition function of the system in matrix form.
   (c) Evaluate the free energy of the system in terms of equilibrium temperature T when the interaction between the spins are switched off (i.e. $I \to 0$).
   (d) If the total magnetic moment of the system is denoted as D then prove that $D \propto \tanh \beta\mu B$ in the limit $I \to 0$ where $\beta^{-1} = k_B T$ with k_B is the Boltzmann constant.

[1+5+2+2=10]

5. (a) What do you mean by 'density matrix' in quantum statistics? For a time dependent density matrix can we have the density matrix as a function of energy eigenvalues of respective basis states by which a quantum mechanical system is represented?
   (b) Lets take 1-D ising model with N spins at very low temperature when almost all the spins are aligned parallel to each other in the same direction. Now, if there is a flip

in spin then it costs an energy 2ϵ. In a configuration with r spin flips the energy is given as

$$E_r = -2\epsilon + 2r\epsilon,$$

where $r = 0, 1$ and 2 with degeneracy ${}^2C_r$ for the r'th spin flips. Find the partition function of the system in canonical ensemble. The symbols carry their usual meanings.

(c) A 1-D chain is hung vertically from a ceiling. One of its extremes is fixed at one end while the other end holds a mass M in presence of gravity. Assume that the gravity is acting along $-\mathbf{z}$ and the chain is formed by two kinds of distinguishable rings which are ellipses in shape with major and minor axes are respectively as α and β. If the total number of rings is fixed at N with equilibrium temperature T then find the average energy and average length of the chain.

[(1+2)+2+5=10]

6. (a) How can you differentiate a weakly degenerate ideal Fermi gas with a strongly one with respect to the most probable particle number distribution for the mentioned gas?

(b) BE condensation happens for ${}_2He^4$ liquid kept at ambient pressure at temperature $T = 2.17$K. At what temperature ${}_2He^4$ gas will be at condensation stage for which the density is 10^3 times smaller than the liquid case?

(c) The general form of Saha ionization equation is given as

$$\frac{n_{i+1}n_e}{n_i} = \frac{g_{i+1}}{g_i}\frac{(2\pi m k_B T)^{3/2}}{h^3}e^{-\frac{\chi}{k_B T}}.$$

Now defining HI, HII respectively as neutral hydrogen and ionized hydrogen along with the number densities for total hydrogen atoms, neutral atoms and ions respectively as n_H, n_{HI} and n_{HII}, establish the following relation for the degree of ionization $x\,(= n_{HII}/n_H)$

$$\frac{x^2}{1-x} \propto \frac{1}{n_H\lambda^3}e^{-\frac{\chi}{k_B T}},$$

where $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$. What can you infer about the state of ionization for a star if the proportionality constant becomes one at very high temperature with $\frac{1}{n_H\lambda^3} \to 10^{-5}$? The symbols carry their usual meanings here.

[2+2+(5+1)=10]

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M.Sc. (1st Year) 2nd Semester Examination, 2022

Subject: Physics

Paper: PHY/CC/TH/203

General Electronics

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks.

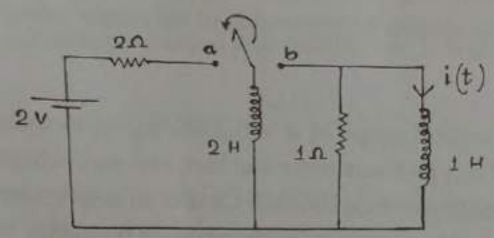
Answer any two questions from each group

*(Use **separate** answer scripts for each group)*

Group-A

1. (a) Draw sketches to show the construction and equivalent circuit of a uni-junction transistor (UJIT). Explain its working principle with the characteristic curve.

(b) The circuit in the fig. was initially in steady state with switch 'S' is position 'a' at t=0. The switch goes from 'a' to 'b' position at t>0. Find **i(t)** for the following network using Laplace transforms at t>0.



(1+1+4) + 4 = 10

2. (a) Show that the diffusion capacitance of a step-graded p-n junction is given by

$C_T = \varepsilon A\sqrt{\dfrac{eN_d}{2\varepsilon\left(V_B + |V_A|\right)}}$ Where N_d is the donor ion concentrations, V_B is the barrier potential and VA is the applied voltage.

(b) What is a TRIAC? Explain its working principle with the characteristic curve.

4 + (1+5) = 10

3. (a). Obtain the expression of voltage gain of an RC coupled amplifier in the mid frequency range.

(b). The mid band gain of an RC coupled amplifier is 120. At frequencies 100 Hz and 100 kHz the gain falls to 60. Determine the lower and upper half power frequencies.

(c). What do you mean by positive and negative feedback amplifier?

(d). The open loop gain of an amplifier changes by 20% due to changes in the parameters of the active amplifying device. If a change of gain by 2% is allowable, what type of feedback has to be applied? If the amplifier gain with feedback is 10, find the minimum value of the feedback ratio and the open loop gain. 3+2+2+3 = 10

Group - B

(Answer any two)

4. (a) Draw the circuit diagram of a J-K flip-flop.
A 4-bit Encoder is constructed in such a way that it can be used as a positive as well as a negative logic system. If in both cases the output is: $Y_0=0$, $Y_1=1$, $Y_2=0$, $Y_3=0$. What is(are) the output(s) in decimal number?
(b) Draw the circuit diagram of an astable multivibrator and obtain an expression of its frequency.
(2+2) + (2+4) = 10

5. (a) Draw the circuit diagram of a 4-to-1 MUX. Construct an XOR gate from it.
(b) Using op-amp, deduce an expression of the output voltage of a non-inverting adder.
(2+2) + 6 = 10

6. (a) Give the circuit diagram of a weighted digital-to-analog converter. What is the disadvantage of this D/A converter and how can we overcome this?
(b) What is amplitude modulation? Obtain an expression of the total power of a modulated wave. Mention one disadvantage of frequency modulation.
(2+1+1) + (1+4+1) = 10

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M.Sc. (1st Year) 2nd Semester Examination, 2022

Subject: Physics

Paper: Electrodynamics

PHY/CC/TH/204

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Answers to the questions should be written in the candidates own words as far as practicable.

Use separate answer scripts for Group A and Group B.

Group – A

Answer any two questions.

1. (a) Starting from the following retarded solution to the 3-D wave equation

$$\Psi(\mathbf{x}, t) = \int \frac{[f(\mathbf{x}', t')]_{ret}}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

show that the source $f(\mathbf{x}', t') = \delta(x')\delta(y')\delta(t')$, equivalent to a point source at the origin with $t = 0$ in 2-D, produces a 2-D wave

$$\Psi(x, y, t) = \frac{2c\Theta(ct - \rho)}{\sqrt{c^2t^2 - \rho^2}},$$

where $\rho^2 = x^2 + y^2$ and $\Theta(\xi = ct - \rho) = 0(1)$ if $\xi < (>)0$. Note that

$$\delta(f(p)) = \sum_i \frac{\delta(p - p_i)}{\left|\left(\frac{df}{dp}\right)_{p=p_i}\right|},$$

where p_i's are the roots of the equation $f(p) = 0$. The symbols carry their usual meanings.

(b) The complex propagation vector for an electromagnetic wave in a media, where the refractive index n is a function of the relative permittivity ϵ_r, is given by

$$\tilde{k} = \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right],$$

where the symbols carry their usual meanings. Evaluate the phase and group velocity of the wave for negligible damping ($\gamma_j = 0$) and comment on your result.

[6+4=10]

2. (a) The diffusion equation for the vector potential $\mathbf{A}$ is

$$\nabla^2\mathbf{A} = \mu\sigma\frac{\partial \mathbf{A}}{\partial t}.$$

The solution of the above equation can be taken as

$$\mathbf{A}(\mathbf{x}, t) = \int d^3x' G(\mathbf{x} - \mathbf{x}', t)\mathbf{A}(\mathbf{x}', 0),$$

where $\mathbf{A}(\mathbf{x}', 0)$ is the initial field configuration and G is an appropriate kernel. Solve the above initial value problem by the use of 3-D Fourier transform in space for $\mathbf{A}(\mathbf{x}, t)$ and show that the kernel takes the form as follows:

$$G(\mathbf{x} - \mathbf{x}', t) = \frac{1}{(2\pi)^3}\int d^3k \, e^{-\frac{k^2 t}{\mu\sigma}} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

The symbols carry their usual meanings.

(b) Consider an infinite parallel plate capacitor with the lower plate (at $z = -d/2$) carrying the charge density $-\sigma$ and the upper plate (at $z = +d/2$) carrying the charge density $+\sigma$. Determine the Maxwell's stress tensor in matrix form. Also determine the momentum per unit area per unit time crossing the $x - y$ plane. [Hint:

$$\overleftrightarrow{T}_{\alpha\beta} = \epsilon_0\left[E_\alpha E_\beta + c^2 B_\alpha B_\beta - \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{E} + c^2\mathbf{B}\cdot\mathbf{B}\right)\delta_{\alpha\beta}\right],$$

where the symbols carry their usual meanings.

[6+4=10]

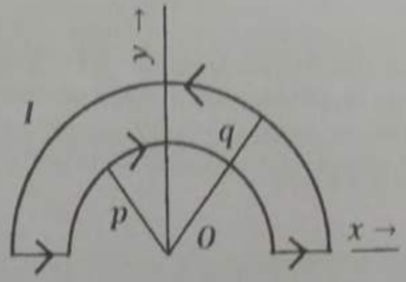

Figure 1:

3. (a) A piece of wire, bent into a loop as shown in Figure (1), carries a current $I(t) \propto t$, for time $t > 0$. Calculate the retarded vector potential ($\mathbf{A}$) at the origin O. Can you determine the magnetic field from the expression obtained for $\mathbf{A}$?

   (b) Consider a rectangular wave guide with dimensions 2.28 cm × 1.01 cm. What TE modes will propagate in this wave guide, if the driving frequency is 1.70×10^{10} Hz?

[6+4=10]

Group – B

Answer any two questions.

4. (a) Suppose a point charge q is moving with uniform velocity along the x direction. Obtain expressions for the Lienard-Wiechert potentials.

   (b) Show that for a Lorentz boost in an arbitrary direction defined by the velocity $\vec{v} = c\vec{\beta}$ the transformations of the electric and magnetic fields are given by

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta} \times c\vec{B}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E}), \quad c\vec{B}' = \gamma(c\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot c\vec{B}).$$

[6+4=10]

5. (a) The source free Lagrange density for the electromagnetic field A_μ is $\mathcal{L} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$, where, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, denotes the electromagnetic field tensor. (i) Derive the corresponding Euler-Lagrange equation of motion. (ii) Show that in terms of the electric and magnetic fields the Hamiltonian density $\mathcal{H}$ is given by $\mathcal{H} = \frac{\epsilon_0}{2}(E^2 + c^2B^2)$.

(b) The four-potential in a certain frame of reference K is given by $A^\mu = (-\frac{E}{c}y, -\frac{B}{2}y, \frac{B}{2}x, 0)$. (i) Calculate the electric and magnetic fields in the reference frame K. (ii) Next consider a Lorentz boost along the x-direction with velocity $\vec{v} = \frac{E}{B}\hat{i}$. Find the transformed four-potential A'^μ in the new frame K'. (iii) Hence deduce expressions for the transformed electric and magnetic fields in the K' frame.

[5+5=10]

6. (a) For a point charge q undergoing acceleration the energy radiated per unit solid angle per unit 'time-at-particle' is given by

$$\frac{dP'}{d\Omega_s} = \frac{\kappa}{\mu_0 c}\left(\frac{q}{4\pi\epsilon_0}\right)^2 \left|\left[\frac{1}{c\kappa^3}\hat{R} \times \{(\hat{R} - \vec{\beta}) \times \dot{\vec{\beta}}\}\right]\right|^2 .$$

Here $R = |\mathbf{r} - \mathbf{x}(t)|$ where $\mathbf{x}(t)$ denotes the position vector of the moving charge and $\kappa = 1 - \hat{R}\cdot\vec{\beta}$, while the notation $[...]$ means that the contents of the square bracket are to be evaluated at the retarded time. The other symbols have their usual meanings.

i. Show that when the acceleration of the charge is parallel to the velocity then

$$\frac{dP'}{d\Omega_s} = \frac{q^2}{4\pi\epsilon_0}\frac{\dot{\beta}^2}{4\pi c}\left[\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}\right].$$

ii. If the direction in which $\frac{dP'}{d\Omega_s}$ is observed to be a maximum be $\theta = 60^\circ$ find β.

(b) A circular ring of radius a with linear charge density λ per unit length lies in the $x-y$ plane. Show that the potential at any point in space for $r \geq a$ is given by

$$\psi(r,\theta,\phi) = \frac{\lambda}{2\epsilon_0}\sum_{n=0}^{\infty}(-1)^n \frac{\left(\frac{1}{2}\right)_n}{n!}\left(\frac{a}{r}\right)^{2n+1} P_{2n}(\cos\theta),$$

where $(p)_n = p(p+1)\cdots(p+n-1)$ denotes the Pochhammer symbol.

[6+4=10]