DHWU

M.Sc. (1st Year) 1st Semester Examination, 2022

Subject: Physics

Paper: Mathematical Methods

PHY/CC/TH/101

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Answers to the questions should be written in the candidates own words as far as practicable.

Group – A

Answer any two questions.

1. (a) Expand the function $\sin \pi x$ in a series of function ϕ_i that are orthogonal in the range $0 \leq x \leq 1$ when the scalar product is defined as

$$< f|g >= \int_0^1 f^*(x)g(x)dx.$$

Keep the first four terms of the expansion. The first four ϕ_i's are $\phi_0 = 1$, $\phi_1 = 2x - 1$, $\phi_2 = 6x^2 - 6x + 1$ and $\phi_3 = 20x^3 - 30x^2 + 12x - 1$.

(b) A 2-D manifold (subspace) in 3-D space is defined by the two vectors $\hat{\chi}_1 = \hat{i} + \hat{j} + 2\hat{k}$ and $\hat{\chi}_2 = -\hat{i} + 2\hat{j} - 3\hat{k}$. Construct the orthonormalized states from them via Gram-Schmidt orthogonalization method.

[7+3=10]

2. (a) Use the method of Green's function to find the solution of $y'' - k^2 y = x$ for $x \in (-\infty, +\infty)$ and satisfying the boundary conditions $y(\pm\infty) = 0$.

(b) Obtain a particular solution of the differential equation

$$y'' + 4y = cosec 2x,$$

by the method of variation of parameters.

[5+5=10]

3. (a) Show that $z = \infty$ is a regular singular point of the equation,

$$z(1-z)\frac{d^2w}{dz^2} + [c - (a+b+1)z]\frac{dw}{dz} - abw = 0,$$

where a, b and c are constants. Determine the values of the corresponding indicial exponents.

(b) Find the eigenvalues and corresponding normalized eigenvectors of the following matrix

$$A = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}.$$

[6+4=10]

Group – B

Answer any two questions.

4. (a) Show that $\mathcal{A}\frac{d^n}{dx^n}(x^2-1)^n$ is the solution of the Legendre's equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

Thereafter, find the value of the constant $\mathcal{A}$.

(b) Consider $f(z) = u + iv$ where

$$f(z) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} + i\frac{x^3+y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}.$$

Show that f is not complex differentiable at $z = 0$, even though the Cauchy-Riemann equations are satisfied at $z = 0$.

[(4+2)+4=10]

5. (a) Identify the branch points of $f(z) = \sqrt{z-1}$ and indicate a suitable branch cut on the complex plane.

(b) Find a Laurent series expansion of the function $f(z) = \frac{z}{z^2+3z+2}$ in the region $1 < |z| < 2$.

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(c) Construct an analytic function $f(z)$ whose real part is $e^{-x}(x\cos y + y\sin y)$ and for which $f(0) = 1$. [3+3+4=10]

6. Evaluate any two of the following:

(a) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

(b) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$, where a and b are positive constants.

(c) $\int_0^{\infty} \frac{\sin px}{x} dx$ for any $p > 0$. [2×5=10]

DHWU

M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2022

Subject : Physics

Paper: Phy/CC/Th/102

Classical Mechanics

Time : 2 Hours

Full Marks : 40

The figures in the margin indicate full marks.

(Candidates are to write their answers in their own words as far as practicable.)

Group-A

*Attempt any **two** questions.*

1. (a) Deduce the Euler-Lagrange equations for the following functional assuming end-point variations to vanish

$$J[y_1, y_2] = \int_{x_0}^{x_1} \left(\frac{1}{2} y_1''^2 - a y_1 y_1'^2 + b y_1^4 + \frac{1}{2} y_2'^2 - c(y_1 + y_2) y_2 \right) dx,$$

where a, b and c are constants. 3

(b) The Hamiltonian of a system is

$$H = \frac{p^2}{2m} - bqpe^{-\alpha t} + \frac{bm}{2} q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{k}{2} q^2.$$

Find a Lagrangian corresponding to this Hamiltonian. Show that there exists an equivalent Lagrangian that is not explicitly time dependent. 4

(c) A bead of mass m slides without friction on the curve $y = \sqrt{x}$ under the action of gravity. Find the equation of motion by using a Lagrange multiplier. 3

2. (a) Prove that Hamilton's equations will remain invariant under any transformation whose Jacobian $\mathcal{T}$ satisfies $\mathcal{T} J \mathcal{T}^T = J$ where J denotes the symplectic matrix. 3

(b) Find the canonical transformation corresponding to the generating function $F_3(q, P) = -(e^Q - 1)^2 \tan p$. 2

(c) In a four-dimensional phase space (q_1, q_2, p_1, p_2) if

$$X_1 = \frac{1}{4}\left[(p_1^2 + q_1^2) - (p_2^2 + q_2^2)\right], \quad X_2 = \frac{1}{2}(p_1 p_2 + q_1 q_2), \quad X_3 = \frac{1}{2}(q_1 p_2 - q_2 p_1),$$

show that their Poisson brackets obey the algebra $\{X_i, X_j\} = \epsilon_{ijk} X_k$ with $(i, j, k = 1, 2, 3)$ and ϵ_{ijk} being the completely anti-symmetric Levi-Civita tensor. 5

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3. (a) Find under what conditions

$$Q = \frac{\alpha p}{q}, \quad P = \beta q^2$$

where α and β are constants, represents a canonical transformation for a system with one degree of freedom. 2

(b) The relativistic Lagrangian of a conservative one particle system is $L = -m_0 c^2 \sqrt{1 - \vec{v}^2/c^2} - U(\vec{x})$. (i) Deduce an expression for the corresponding Hamiltonian. (ii) Assuming a Kepler potential i.e., $U(\vec{x}) = -\lambda/r$ with $\lambda > 0$ and $r^2 = \vec{x} \cdot \vec{x}$ show that the orbit is determined by

$$\frac{dr}{d\theta} = \frac{r^2}{p_\theta} \sqrt{\left(E + \frac{\lambda}{r}\right)^2 - m_0^2 c^4 - \frac{c^2 p_\theta^2}{r^2}},$$

where E represents the energy of the system and p_θ is a conserved quantity. (iii) Use the transformation $u = 1/r$ to show that the equation for the orbit may be expressed by

$$\frac{d^2 u}{d\theta^2} = (b^2 - 1)u + ab, \quad \text{where } b = \frac{\lambda}{c p_\theta}, \quad a = \frac{E}{c p_\theta}.$$

(iv) Hence show that for $b^2 < 1$

$$\frac{1}{r} = A \cos(\sqrt{1 - b^2}(\theta - \theta_0)) + \frac{ab}{1 - b^2}$$

where A and θ_0 are arbitrary constants. 2+3+2+1=8

Group-B

*Attempt any **two** questions.*

4. (a) A very thin rectangular sheet of uniform density and sides a and $2a$ rotates freely in space.
(i) Find the moments of inertia along the three principal axes passing through its center of mass.
(ii) Obtain the Euler equations for this system.
(iii) Discuss qualitatively the subsequent rotational motion of the object in the three cases when the initial angular velocity $\vec{\omega}$ is almost, but not exactly parallel to each of the principal axes. (3 + 2) + 5 = 10

5.(a) A rigid body, fixed in three dimensions, undergoes three successive rotations by angles $(\pi/2, \pi, \pi/2)$, respectively, along (z, x, z) symmetry axes. Obtain the transformation matrix A resulting from three rotations using matrix operations. Find the symmetry axis and the angle of rotation, if any, under $\vec{X}' = A\vec{X}$. 4

(b) A $1kg$ top has center of gravity at point G. If it spins about its axis of symmetry and precesses about the vertical axis at constant rate of $\omega_s = 60\ rad/s$ and $\omega_p = 10\ rad/s$, respectively, determine the steady state angle θ. The radius of gyration of the top about the $z-$ axis is $k_z = 1\ cm$ and about the x and y axes are $k_x = k_y = 4\ cm$. Deduce the relation required for solving the problem.
The kinetic energy and the potential energy of the heavy symmetric top are given by: $T = \frac{1}{2}I_1(\dot{\theta}^2 + \dot{\phi}^2 sin^2\theta) + \frac{1}{2}I_3(\dot{\psi} + \dot{\phi}cos\theta)^2$ and $V(x) = Mglcos\theta$. (Here notations have their usual meanings) 6

6. (a) Show that $A_{\mu\nu}B^{\nu\alpha}$ is a mixed tensor of rank 2. 2

(b) Show that the D'Alembertian operator $\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ remains invariant under Lorentz transformation. 3

(c) Define 4-velocty vector. Hence show that 4-velocity vector is orthogonal to the Minkowski 4-force vector. 2

(d) Two rockets of rest length L_0 are approaching the earth from opposite directions at velocities $\pm\frac{c}{2}$. How long does one of them appear to the other? 3

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M.Sc. (1st Year) 1st Semester Examination, 2022
Subject: Physics
Paper: PHY/Th/1S/103
Quantum Mechanics I

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Answer two questions from each group
*(Use **separate** answer script for each group)*

Group-A

1. (a) For a LHO with Hamiltonian $H=\frac{p^2}{2m}+\frac{1}{2}kx^2$, show that $H=\hbar\omega(a_+a_-+\frac{1}{2})$, where a_- and a_+ are lowering and raising operators.

Using a_-, find the ground state wave function of the LHO. Hence find its wave function of 1st excited state.

(b) Find the expectation value of the potential energy in the nth state of the harmonic oscillator. (3+2+2)+3=10

2. (a) For an eigenstate of J^2 and J_z, show that

$$\langle J_x\rangle=\langle J_y\rangle=0$$
$$\langle J_x^2\rangle=\langle J_y^2\rangle=\frac{1}{2}\hbar^2[j(j+1)-m^2]$$

(b) Obtain the matrix of Clebsh-Gordan coefficient for addition of angular momenta $j_1=1$ and $j_2=1/2$. (2+3) +5=10

3. (a) The rotational part of a Hamilonian of a diatomic molecule is $\frac{1}{2I}(L_x^2+L_y^2)+\frac{1}{I}L_z^2$, I is the moment of inertia. Find the energy eigen values.

(b) Is J_1-J_2 an angular momentum operator?

(c) Prove that

$[XP_x,H]=\frac{i\hbar}{m}P_x^2+X[P_x,V]$ where $H=\frac{P_x^2}{2m}+V$, and V is the potential energy operator.

(d) Show that commutating operators have common set of eigenfunctions.

3+2+3+2=10

Please turn over.

Group-B

4. (a) Show that in interaction picture, time evolution of the wave function is governed by interaction Hamiltonian in interaction representation.
What are the basic features of the interaction picture in quantum mechanics?

(b) Using variational method, find the ground state energy of Helium atom. Given $<1/r_{12}> = \frac{5}{4}Z_{eff}E_H$ and E_H=-13.6 eV.

(c) Obtain energy eigen values of a 1D harmonic oscillator using WKB approximation.

(2+1)+5+2=10

5. (a) Calculate the 1st order correction to the ground state energy of a 1D anharmonic oscillator of mass m and angular frequency ω subject to a potential $V(x)= \frac{1}{2} m\omega^2x^2 + bx^4$. Given the ground state wave function as

$$\psi_0^{(o)} = (m\omega/\pi\hbar)^4 exp(-m\omega x^2/2\hbar).$$

Treat bx^4 as the perturbation.

(b) A rigid rotator having a moment of inertia I and electric dipole moment μ executes rotational motion in a plane. Estimate the 2nd order correction to energy when the rotator is acted upon by an electric field E in the plane of rotation. Given $E_n^0 = \frac{n^2\hbar^2}{2I}$.

(c) A particle is trapped in an infinite square well of width b and its unperturbed wave function is given $\psi_n^{(o)} = \sqrt{\frac{2}{b}} \sin\left(\frac{n\pi x}{b}\right)$. If the system is perturbed by raising the floor by a constant potential V_0, find the 1st order and 2nd order corrections to the energy in its n^{th} state.

3+4+3=10

6. (a)Treating relativistic interaction as $H' = -\frac{[E_n - V(r)]^2}{2m_0c^2}$ obtain an expression of its effect on the energy level of an one-electron system. The terms in H' have the usual meanings.

(b) Obtain an expression of WKB wave function for a particle with E > V.

(c) Obtain the classical turning point (s) of a one-dimensional harmonic oscillator.

4+4+2=10

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M.Sc. 1st Semester Examination, 2022

Subject: Physics

Paper: PHY/CBCS/1S/104 (Numerical Methods & Computer Programming)

Time: 3 Hours **Full Marks: 35**

*Answer any **one** question from each group*

Answers to the questions should be written in the candidates own words as far as practicable.

(Use separate answer scripts for each group)

Marks Distribution: LNB - 5, Viva - 10, Exam - 17.5 * 2=35 (Total=50)

Group – A

Instructions:

Qs.**1** for Roll nos. 1-12; Qs.**2** for Roll nos. 13-25, Qs.**3** for Roll nos. 26-41; Qs.**4** for Roll nos. 34-53

Exam (Gr – A, Marks-17.5): (i) Working formula - **2**, (ii) Writing program in **F95** - **10**, (iii) Skill - **4** (input & output using files, different accuracy/divisions, formatting, etc.), (iv) Comments - **1.5** (only one comment on accuracy/convergence).

3. A real root using **Newton-Raphson** method: $f(x) = x + \log_{10}x - 2 = 0$.

OR, Integration using composite **Trapezoidal** rule: $I = \int_0^1 \frac{\ln(1+x^2)}{1+x^2}\, dx$

4. A real root using **Bisection** method: $f(x) = \log_e x - \cos x = 0$.

OR, Integration using composite **Simpson's 1/3** rule: $I = \int_0^1 \frac{\ln(1+x^2)}{1+x^2}\, dx$

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M.Sc. 1st Semester Examination, 2022

Subject: Physics

Paper: PHY/CBCS/TS/104 (Numerical Methods & Computer Programming)

Time: 3 Hours | Full Marks: 35

*Answer any **one** question from each group*

Answers to the questions should be written in the candidates own words as far as practicable.

(Use separate answer scripts for each group)

Marks Distribution: LNB - 5, Viva - 10, Exam - 17.5 * 2=35 (Total=50)

Group – B

Instructions:

Qs.1 for Roll nos. 1-12; Qs.2 for Roll nos. 13-25, Qs.3 for Roll nos. 26-41; Qs.4 for Roll nos.42-53

Exam (Gr - B, Marks - 17.5): (i) Working formula/Graph - 2, (ii) Program writing in F95 - 10, (iii) Skill - 4 (data input - output by using files/ accuracy/ innovative techniques/ compilation by self-etc.), (iv) Comments/ Discussion - 1.5.

3. Estimate the value of 'π' numerically via the area evaluation concept with the help of Monte Carlo simulation by the generation of random numbers.

4. Find the area under the curves $y = x, x = 1$ and $y = 0$ via Monte Carlo simulation with the generation of random numbers.