**DHWU**
**M.Sc. (1st Year) 2nd Semester Evaluation, 2021**
**Subject: Physics**
**Paper: PHY/CC/TH/201**
**Quantum Mechanics II**

**Full Marks: 40** **Time: 2 Hours**

*The figures in the margin indicate full marks.*
***Answer any two questions from each group***
*(Use separate answer scripts for each group)*

## Group-A

1. a) Show that the fundamental commutation relation $[x_i, p_j] = i\hbar\delta_{ij}$ is invariant under time reversal symmetry

   b) Consider rotation about an arbitrary axis denoted by $R(\phi, \hat{n}) = e^{-\frac{i}{\hbar}\phi\hat{n}\cdot\vec{J}}$ where $\hat{n}$ denotes the arbitrary direction of the axis and $\phi$ stands for the angle of rotation about $\hat{n}$. Construct the rotation matrix for spin ½ system. What group does the system correspond to?
   What happens to a spin ½ particle under $2\pi$ rotation? In what respect does the group differ from $SO(3)$ group?

   c) Express $xz$ in terms of components of a spherical 2nd rank tensor (irreducible).

   2 + (3+1+1+1) + 2 = 10

2. Consider an electron moving under the influence of a periodic potential, $V(x + a) = V(x)$ in one dimension where $a$ is the lattice spacing of atoms.

   a) Show that the translational symmetry operator commutes with the Hamiltonian of the system. Show that the plane wave solution is an eigenstate of the translation operator.

   b) Is the eigenstate of the Hamiltonian the same as that of the translation operator? Explain.

   c) Assuming $|n\rangle$ be the eigenket at $n^{th}$ site of the lattice with the only bound state satisfying, $H|n\rangle = E_0|n\rangle$, show that $|n\rangle$ is not an eigenstate of $T(a)$. Is there any assumption on barrier height of the potential? Construct a state which is simultaneous eigenstate of $H$ and $T(a)$. Explain the degeneracy associated with the problem.

   d) What is "Tight binding approximation"? Under this condition show that degeneracy is lifted with continuous energy band. Draw the energy band diagram.

   2 + 1 + 4 + 3 = 10

3. a) Obtain Dirac equation in covariant form starting from the equation
$(\vec{\alpha}\cdot\vec{p}+\beta m)\psi = i\hbar\frac{\partial\psi}{\partial t}$ where, notations have their usual meanings.
b) Derive the equation of continuity in this context. Interpret your result physically.
d) Consider transition between states of a quantum system triggered by a vector operator designated by $T(1,q)$ where, $q = 0, \pm 1$. Show that allowed transitions are possible for $\Delta q = 0, \pm 1$.
e) Suppose we have two spin ½ uncoupled states. How many states do we have after combining them by tensor product? Identify the reducible and irreducible states.
3 + 3 + 2 + 2 = 10

## Group – B

(Answer any two)

4. a) Write down all possible wave functions (spin-orbitals) of the ground and 1st excited state of He atom

b) Find the total scattering cross section of a particle, in the low energy limit, by a square well potential of the form

$$V(r) = -V_0, \text{ for } 0 < r < a$$
$$= 0, \text{ for } r \geq a, \text{ a=width of the potential well.}$$

5+5=10

5. a) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, for a constant perturbation: $H'(t') = H'$ for $0 \leq t' \leq t$ ; zero otherwise, obtain Fermi Golden rule for the transition rate from a discrete state i to a continuum state k.

b)Using this Fermi golden rule obtain an expression for differential scattering cross section of a particle by spherically symmetric potential.
5+5=10

6. a) Show that for scattering of a particle by a spherically symmetric potential, the asymptotic solution of the Schrodinger equation can be taken of the following form (terms having their usual meanings): $\Psi(\mathbf{r}) = e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$.

b) Obtain the relation between differential scattering cross section and scattering amplitude f(ϑ). How will the expression be modified for indistinguishable particles?
5+(4+1)=10

Suyo Pa~

**1×1**

| $m_1$ | $m_2$ | 2, +2 |
|---|---|---|
| +1 | +1 | 1 |

| $m_1$ | $m_2$ | 2, +1 | 1, +1 |
|---|---|---|---|
| +1 | 0 | 1/2 | 1/2 |
| 0 | +1 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 2, 0 | 1, 0 | 0, 0 |
|---|---|---|---|---|
| +1 | −1 | 1/6 | 1/2 | 1/3 |
| 0 | 0 | 2/3 | 0 | −1/3 |
| −1 | +1 | 1/6 | −1/2 | 1/3 |

| $m_1$ | $m_2$ | 2, −1 | 1, −1 |
|---|---|---|---|
| 0 | −1 | 1/2 | 1/2 |
| −1 | 0 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 2, −2 |
|---|---|---|
| −1 | −1 | 1 |

**1/2×1/2**

| $m_1$ | $m_2$ | 1, +1 |
|---|---|---|
| +1/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | 1, 0 | 0, 0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | 1/2 |
| −1/2 | +1/2 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 1, −1 |
|---|---|---|
| −1/2 | −1/2 | 1 |

**1/2×1/2**

| $m_1$ | $m_2$ | 1, +1 |
|---|---|---|
| +1/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | 1, 0 | 0, 0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | −1/2 |
| −1/2 | +1/2 | 1/2 | 1/2 |

| $m_1$ | $m_2$ | 1, −1 |
|---|---|---|
| −1/2 | −1/2 | 1 |

# 34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

| | | $J$ | $J$ | … |
|---|---|---|---|---|
| | | $M$ | $M$ | … |
| $m_1$ | $m_2$ | Coefficients | | |
| $m_1$ | $m_2$ | | | |
| ⋮ | ⋮ | | | |

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

**1/2×1/2**

| $m_1$ | $m_2$ | 1, +1 |
|---|---|---|
| +1/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | 1, 0 | 0, 0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | 1/2 |
| −1/2 | +1/2 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 1, −1 |
|---|---|---|
| −1/2 | −1/2 | 1 |

**2×1/2**

| $m_1$ | $m_2$ | 5/2, +5/2 |
|---|---|---|
| +2 | +1/2 | 1 |

| $m_1$ | $m_2$ | 5/2, +3/2 | 3/2, +3/2 |
|---|---|---|---|
| +2 | −1/2 | 1/5 | 4/5 |
| +1 | +1/2 | 4/5 | −1/5 |

| $m_1$ | $m_2$ | 5/2, +1/2 | 3/2, +1/2 |
|---|---|---|---|
| +1 | −1/2 | 2/5 | 3/5 |
| 0 | +1/2 | 3/5 | −2/5 |

| $m_1$ | $m_2$ | 5/2, −1/2 | 3/2, −1/2 |
|---|---|---|---|
| 0 | −1/2 | 3/5 | 2/5 |
| −1 | +1/2 | 2/5 | −3/5 |

| $m_1$ | $m_2$ | 5/2, −3/2 | 3/2, −3/2 |
|---|---|---|---|
| −1 | −1/2 | 4/5 | 1/5 |
| −2 | +1/2 | 1/5 | −4/5 |

| $m_1$ | $m_2$ | 5/2, −5/2 |
|---|---|---|
| −2 | −1/2 | 1 |

**1×1/2**

| $m_1$ | $m_2$ | 3/2, +3/2 |
|---|---|---|
| +1 | +1/2 | 1 |

| $m_1$ | $m_2$ | 3/2, +1/2 | 1/2, +1/2 |
|---|---|---|---|
| +1 | −1/2 | 1/3 | 2/3 |
| 0 | +1/2 | 2/3 | −1/3 |

| $m_1$ | $m_2$ | 3/2, −1/2 | 1/2, −1/2 |
|---|---|---|---|
| 0 | −1/2 | 2/3 | 1/3 |
| −1 | +1/2 | 1/3 | −2/3 |

| $m_1$ | $m_2$ | 3/2, −3/2 |
|---|---|---|
| −1 | −1/2 | 1 |

**3/2×1/2**

| $m_1$ | $m_2$ | 2, +2 |
|---|---|---|
| +3/2 | +1/2 | 1 |

| $m_1$ | $m_2$ | 2, +1 | 1, +1 |
|---|---|---|---|
| +3/2 | −1/2 | 1/4 | 3/4 |
| +1/2 | +1/2 | 3/4 | −1/4 |

| $m_1$ | $m_2$ | 2, 0 | 1, 0 |
|---|---|---|---|
| +1/2 | −1/2 | 1/2 | 1/2 |
| −1/2 | +1/2 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 2, −1 | 1, −1 |
|---|---|---|---|
| −1/2 | −1/2 | 3/4 | 1/4 |
| −3/2 | +1/2 | 1/4 | −3/4 |

| $m_1$ | $m_2$ | 2, −2 |
|---|---|---|
| −3/2 | −1/2 | 1 |

**2×1**

| $m_1$ | $m_2$ | 3, +3 |
|---|---|---|
| +2 | +1 | 1 |

| $m_1$ | $m_2$ | 3, +2 | 2, +2 |
|---|---|---|---|
| +2 | 0 | 1/3 | 2/3 |
| +1 | +1 | 2/3 | −1/3 |

| $m_1$ | $m_2$ | 3, +1 | 2, +1 | 1, +1 |
|---|---|---|---|---|
| +2 | −1 | 1/15 | 1/3 | 3/5 |
| +1 | 0 | 8/15 | 1/6 | −3/10 |
| 0 | +1 | 2/5 | −1/2 | 1/10 |

| $m_1$ | $m_2$ | 3, 0 | 2, 0 | 1, 0 |
|---|---|---|---|---|
| +1 | −1 | 1/5 | 1/2 | 3/10 |
| 0 | 0 | 3/5 | 0 | −2/5 |
| −1 | +1 | 1/5 | −1/2 | 3/10 |

| $m_1$ | $m_2$ | 3, −1 | 2, −1 | 1, −1 |
|---|---|---|---|---|
| 0 | −1 | 2/5 | 1/2 | 1/10 |
| −1 | 0 | 8/15 | −1/6 | −3/10 |
| −2 | +1 | 1/15 | −1/3 | 3/5 |

| $m_1$ | $m_2$ | 3, −2 | 2, −2 |
|---|---|---|---|
| −1 | −1 | 2/3 | 1/3 |
| −2 | 0 | 1/3 | −2/3 |

| $m_1$ | $m_2$ | 3, −3 |
|---|---|---|
| −2 | −1 | 1 |

**3/2×1**

| $m_1$ | $m_2$ | 5/2, +5/2 |
|---|---|---|
| +3/2 | +1 | 1 |

| $m_1$ | $m_2$ | 5/2, +3/2 | 3/2, +3/2 |
|---|---|---|---|
| +3/2 | 0 | 2/5 | 3/5 |
| +1/2 | +1 | 3/5 | −2/5 |

| $m_1$ | $m_2$ | 5/2, +1/2 | 3/2, +1/2 | 1/2, +1/2 |
|---|---|---|---|---|
| +3/2 | −1 | 1/10 | 2/5 | 1/2 |
| +1/2 | 0 | 3/5 | 1/15 | −1/3 |
| −1/2 | +1 | 3/10 | −8/15 | 1/6 |

| $m_1$ | $m_2$ | 5/2, −1/2 | 3/2, −1/2 | 1/2, −1/2 |
|---|---|---|---|---|
| +1/2 | −1 | 3/10 | 8/15 | 1/6 |
| −1/2 | 0 | 3/5 | −1/15 | −1/3 |
| −3/2 | +1 | 1/10 | −2/5 | 1/2 |

| $m_1$ | $m_2$ | 5/2, −3/2 | 3/2, −3/2 |
|---|---|---|---|
| −1/2 | −1 | 3/5 | 2/5 |
| −3/2 | 0 | 2/5 | −3/5 |

| $m_1$ | $m_2$ | 5/2, −5/2 |
|---|---|---|
| −3/2 | −1 | 1 |

**1×1**

| $m_1$ | $m_2$ | 2, +2 |
|---|---|---|
| +1 | +1 | 1 |

| $m_1$ | $m_2$ | 2, +1 | 1, +1 |
|---|---|---|---|
| +1 | 0 | 1/2 | 1/2 |
| 0 | +1 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 2, 0 | 1, 0 | 0, 0 |
|---|---|---|---|---|
| +1 | −1 | 1/6 | 1/2 | 1/3 |
| 0 | 0 | 2/3 | 0 | −1/3 |
| −1 | +1 | 1/6 | −1/2 | 1/3 |

| $m_1$ | $m_2$ | 2, −1 | 1, −1 |
|---|---|---|---|
| 0 | −1 | 1/2 | 1/2 |
| −1 | 0 | 1/2 | −1/2 |

| $m_1$ | $m_2$ | 2, −2 |
|---|---|---|
| −1 | −1 | 1 |

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

DHWU

M.Sc. (1st Year) 2nd Semester Evaluation, 2021

Subject: Physics

Paper: Statistical Mechanics

PHY/CC/TH/202

Time: 2 Hours

Full Marks: 40

*Answer any 4 questions taking two from each group.*

*Answers to the questions should be written in the candidates own words as far as practicable.*

## Group – A

1. (a) State and prove Liouville's theorem. Discuss its physical significance.

   (b) Show that the partition function $Z_N$ for an extreme relativistic gas consisting of $N$-non-interacting molecules with energy-momentum relationship $\epsilon = cp$ is given by

$$Z_N = \frac{1}{N!}\left[8\pi V\left(\frac{kT}{hc}\right)^3\right]^N.$$

   Hence deduce the pressure of the gas.

[(1+4+1)+4=10]

2. (a) Consider $N$ particles each of mass $m$ in a 3D volume $V$ at a temperature $T$. Assume that the particles are non-interacting and distinguishable,

   i. Calculate the canonical partition function for the $N$ particle system.

   ii. Calculate the free energy ($F$) for the system. Is $F$ is extensive quantity? Explain.

   iii. Calculate the entropy for the system.

   (b) Next consider two reservoirs, each having volume $V$, separated by removable plate. Both reservoirs contain $N$ particles. Calculate the change of entropy $S$ of the total system (defined as the difference between the entropies of the final and initial states) when the separating plate is removed. Explain how reversibility of the process violates $2^{\text{nd}}$ law of thermodynamics.

Suja Pal

[(3+(1+2)+1)+(2+1)=10]

3. (a) A system can take only three different energy state $\epsilon_1 = 0$, $\epsilon_2 = 1.38 \times 10^{-21}$ joule, $\epsilon_3 = 2.76 \times 10^{-21}$ joule. These three states can occur in 2, 5 and 4 different way respectively. Find the probability that at temperature 100K the system may be in one of the microstate of energy $\epsilon_3$

(b) For a one dimensional system, show that the density of state (for non-relativistic energy) is given by

$$D(\epsilon) = \frac{g}{2\pi\hbar}\sqrt{\frac{2m}{\epsilon}}.$$

(c) Consider a system of an extreme relativistic gas consisting of $3N$ particles moving in 1 dimension. Show that the partition function is given by $\frac{1}{3N!}\left[2L\left(\frac{kT}{hc}\right)\right]^{3N}$. $L$ being the length of the space available.

[4+3+3=10]

## Group – B

4. (a) Consider an electron is placed in an external magnetic field viz. $\mathbf{B} = B_0\hat{e}_z$, where $B_0$ is constant and $\hat{e}_z$ is the unit vector along $z$. If the Hamiltonian of the system is $H = -\mu_B(\sigma \cdot \mathbf{B})$, then find its density matrix in canonical ensemble. Thereafter, also find $<\sigma_z>$. The symbols carry their usual meanings.

(b) Our universe is filled with black body radiation (photons) at a temperature 3 K. This is thought to be one of the basic phenomenon of the early developments of the universe following the big bang. Express the photon number density in terms of temperature and other universal constants. [Note: A certain numerical cofactor may be left in form of a dimensionless integral in the calculation.]

[(4+2)+4=10]

5. (a) Draw a neat graph for $C_V$ vs $T$ for an ideal Bose gas and discuss the various physical phenomena happens inside the system for temperatures $T < T_0$, $T = T_0$ and $T > T_0$; where $T_0$ is the condensation temperature of the system.

(b) Consider a system of $N$ non-interacting particles at temperature $T$ with each one possesses a vibrational energy $\epsilon = (n + 1/2)\hbar\omega_0$, where $n = 0, 1, 2, 3, ...$ and $\omega_0$ is the natural frequency of vibration of each particle. Find the partition function of this system.

[6+4=10]

6. (a) How can you relate the Saha ionisation to the formation of plasma at earth? Can every ionised gas be labelled with the term 'plasma'? Explain.

   (b) Deduce the pressure of a partially ionised hydrogen gas system in terms of the degree of the ionisation factor.

[(2+2)+6=10]

Suvo Pal.

**DHWU**
**M.Sc. (1st Year) 2nd Semester Evaluation, 2021**
**Subject: Physics**
**Paper: PHY/CC/TH/203**
**General Electronics**

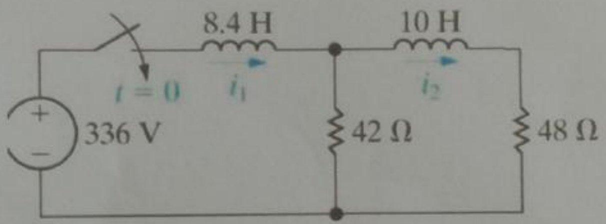
**Full Marks: 40** **Time: 2 Hours**

*The figures in the margin indicate full marks.*
***Answer any two questions from each group***
*(Use* ***separate*** *answer scripts for each group)*

## Group-A

1. (a) Draw sketches to show the construction and equivalent circuit of a uni-junction transistor (UJIT). Explain its working principle with the characteristic curve.

(b) Find $i_1(t)$ and $i_2(t)$ for the following network using Laplace transforms for t>0.




**(1+1+4)+4=10**

2. (a) Obtain an expression for depletion layer width in the context of Depletion Layer Model. Find the limiting expressions for depletion layer width in P+N junction and N+P junction.

(b) What are the factors that affect the bias stability of a transistor? What are the advantages of h parameters?

(c) A CE transistor amplifier is characterised by $h_{ie} = 2K\Omega$, $h_{re} = 2 \times 10^{-4}$, $h_{fe} = 50$, $h_{oe} = 2 \times 10^{-6} A/V$. If the load resistance is 4 KΩ and the source resistance is 200 Ω, determine the input resistance, the output resistance, and the voltage, current and power gains. **4+1+(1+1)+3=10**

3. (a) A BJT is a current controlled device while a FET is a voltage-controlled device. Justify.
(b) An n channel enhancement mode MOSFET shows a saturation drain current of 5 mA for a gate-source voltage of 8V. if the threshold voltage is 4V, what is the saturation drain current for a gate-source voltage of 10V.
(c) What is a tuned amplifier? In which range of frequencies are tuned amplifiers used?
(d) The open loop gain of an amplifier changes by 5%. If 10dB negative feedback is applied calculate the percentage change of the closed loop gain. **2+3+(1+1)+3=10**

Suj Pdw.

## Group - B

4. (a) Design a J-K flip-flop, give its truth table and mention its main advantage.
   (b) Draw the circuit diagram of an astable multivibrator and obtain an expression of its frequency. **(2+1+1)+(2+4)=10**

5. (a) Draw the circuit diagram of a 4-to-1 MUX and construct an exclusive-OR gate from it.
   (b) Using op-amp, deduce an expression of the output voltage of a non-inverting adder. **(3+2)+5=10**

6. (a) Design a first order active low-pass filter at a cut-off frequency 1 kHz with a pass band gain of 2. Deduce the necessary formula from the circuit diagram. Plot the frequency response of this filter.
   (b) Draw the circuit diagram of a logarithmic amplifier using op-amp. **(3+4+1)+2=10**

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Diamond Harbour Women's University
M.Sc 2nd Semester Evaluation 2021
Subject: Physics
Paper: PHY/CC/TH/204 (Electrodynamics)

*Time : 2 hours* *Full Marks: 40.*

*Answer any 4 questions taking two from each group.*
*Answers to the questions should be written in the candidates own words as far as practicable.*

## GROUP A

Q1.(a) Show that the solution of the Dirichlet exterior problem for a infinitely long conducting cylinder of radius $a$ with its axis along the $z$ direction is given by

$$\Phi_{ext}(\rho,\phi) = \frac{1}{2\pi}\int_0^{2\pi} \Phi_s(\phi')\frac{(\rho^2-a^2)d\phi'}{(\rho^2+a^2-2\rho a\cos(\phi-\phi'))}, \quad \rho > a$$

(b) Given

$$\Phi(\rho=a,\phi) = \begin{cases} V, & 0<\phi<\pi \\ -V, & -\pi<\phi<0 \end{cases}$$

prove that the exterior solution is

$$\Phi_{ext}(\rho,\phi) = \frac{V}{\pi}\left[2\tan^{-1}\left(\frac{2a\rho\sin\phi}{\rho^2-a^2}\right)-\pi\right], \quad \rho > a$$

$$\left[\text{Hint: } \int\frac{d\phi'}{\rho^2+a^2-2\rho a\cos(\phi-\phi')} = -\frac{2}{(\rho^2-a^2)}\tan^{-1}\left(\frac{\rho+a}{\rho-a}\tan\left(\frac{\phi-\phi'}{2}\right)\right)\right]$$

[6+4=10]

Q2.(a) Deduce the TE-mode solution for a rectangular wave guide of sides $a$ and $b$ with $a > b$ placed along the $z$ axis.

(b) A point charge $q$ rests at the origin. A natural choice of potentials for this static problem is

$$V(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0 r}, \qquad \mathbf{A}(\mathbf{r},t) = 0$$

Suy5P2

| (V) | | |
|---|---|---|
| 0 | 0 | 0 |
| 1.0 | 30 | 27 |
| 2.0 | 52 | 42 |
| 3.0 | 98 | 63 |
| | | 69 |

of the electric and magnetic fields when the sources are absent. [3+2+5=10]

Q6 (a) An infinite straight wire with charge $\lambda$ per unit length lies stationary along the $x$-axis of an inertial frame $S$. Obtain the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ in an inertial system $S'$ which moves at a constant relativistic velocity $\mathbf{v}$ along the common $x - x'$ axis.

(b) If $\mathbf{E} \cdot \mathbf{B} = 0$ and $|\mathbf{E}| > c|\mathbf{B}|$ find the inertial frame in which the field is (i) purely electric and (ii) purely magnetic.

[6+4=10]

Consider the gauge transformation $\lambda(r,t) = \frac{qt}{4\pi\epsilon_0 r} + \frac{k}{r^3}$, where $k$ is a constant. Calculate the transformed potentials $V'$ and $\mathbf{A}'$ and the fields. [6+4=10]

Q3. (a) The first term in the multipole expansion of the vector potential for a current source $\mathbf{J}(\mathbf{x}, t)$ is

$$A_1(\mathbf{x}, t) = \frac{\mu_0}{4\pi x} \int d^3x' \mathbf{J}(\mathbf{x}', t - \frac{x}{c}).$$

Explain why this term is called the electric dipole term $\mathbf{A}^{ED}(\mathbf{x}, t)$ of the vector potential. Calculate the contribution of $\mathbf{A}^{ED}(\mathbf{x}, t)$ to the magnetic field up to order $1/r$.

(b) An infinite straight wire carrying current, $I(t) = kt\theta(t)$, where $k$ is a constant and $\theta(t) = 1$ for $t > 0$ and zero otherwise, is placed along the $z$ axis. Find the vector potential at a perpendicular distance $s$ from the wire. Calculate the corresponding electrical field. (Assume the wire to be electrically neutral.) [5+ 5=10]

GROUP B

Q4.(a) For radiation from relativistic particles show that when the velocity $v$ is perpendicular to the acceleration $\dot{v}$, the total power radiated is

$$P' = \frac{q^2}{4\pi\epsilon_0}\left(\frac{2}{3c}\right)\left(\frac{\dot{v}}{c}\right)^2 \gamma^4,$$

where the symbols have their usual meanings.

Hint $$\int_{-1}^{1} \frac{2(1-\beta\mu)^2 - (1-\beta^2)(1-\mu^2)}{(1-\beta\mu)^5} d\mu = \frac{8}{3(1-\beta^2)^2}.$$

(b) Find the energy radiated by an electron moving in a circular orbit of radius $R$ with velocity $v$ after one complete revolution in a cyclotron. [7+3=10]

Q5. Derive the field equations from the following Lagrangian density

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu$$

where $A_\mu$ denotes the components of the four-potential and $J^\mu$ represents the four-current density. Using the Lorentz gauge find the equations satisfied by the scalar and vector potentials. Calculate the conjugate momenta and derive an expression for the Hamiltonian in terms