DHWU

M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2021

Subject : Physics

(Phy/CC/Th/101)

Paper Code: Phy/Th/1S/101(*For Supplementary Candidates*)

Mathematical Methods

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks

(*Candidates are required to give their answers in their own words as far as practicable*)

Group-A

Answer any two questions.

1. (a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$. 5

Are the eigenvectors orthogonal?

(b) Show that the following set of vectors in $\mathbb{R}^4$ are linearly independent and orthonormalize them by the Gram-Schmidt method,

$$x_1 = (1,0,1,1), \quad x_2 = (-1,0,-1,1), \quad x_3 = (0,-1,1,1).$$

5

2. (a) State whether the matrix $\begin{pmatrix} a & b \\ 1-a & 1-b \end{pmatrix}$ is a Markov matrix? Find its eigenvalues. Is the product of two such Markov matrices also a Markov matrix? 4

(b) Show that the Fourier transform of a Gaussian function, $f(x) = Ae^{-x^2/2a^2}$, is also a Gaussian function. Prove that the Fourier transform is a unitary operator in the space $L^2(\mathbb{R})$? 6

3. (a) Solve the following system of equations by eigenvalue method

$$\frac{dU_1}{dt} = -U_1 + 2U_2, \quad \frac{dU_2}{dt} = U_1 - 2U_2$$

5

with initial conditions $U_1(0) = 1$ and $U_2(0) = 0$.

5

(b) Obtain the general solution of the equation, $y'' + y = cot x$.

Group-B

Answer any two questions.

5. (a) Show that $z = 0$ is a regular singular point of the equation,

$$z(1-z)\frac{d^2w}{dz^2} + [c - (a+b+1)z]\frac{dw}{dz} - abw = 0,$$

where a, b and c are constants and obtain its general solution about $z = 0$. 6

(b) Prove that

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

(n being a non-negative integer and $|x| < 1$) can be mapped to the Gauss hypergeometric equation by the change of independent variable $t = (1-x)/2$. What are the values of the corresponding parameters of the hypergeometric equation? 4

6. (a) Determine where the function $f(z) = (x + \alpha y)^2 + 2i(x - \alpha y)$ is analytic when α is a real constant. 3

(b) Expand the function $f(z) = \frac{(z-1)^2}{z(z+1)^3}$ in a Laurent series in the region $|z| > 1$. 3

(c) Describe the singularities of $f(z) = \frac{1}{\sin\frac{\pi}{z}}$ and sketch them. State which of these singularities are isolated. 4

7. (a) Suppose $f = u + iv$ is analytic in a rectangle with sides parallel to the coordinate axes and satisfies $u_x + v_y = 0$ for all x, y. Show that there exists a real constant c and a complex constant d such that $f(z) = -icz + d$. 4

(b) Use Cauchy's residue theorem to evaluate

$$I = \int_{-\infty}^{\infty} \frac{x^2}{x^4+1} dx$$

with a semicircular contour of radius R in the upper half plane. 6

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M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2021

Subject : Physics

Paper Code: Phy/CC/Th/102

Paper Code: Phy/Th/1S/102(*For Supplementary Candidates)*

Classical Mechanics

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks

(Candidates are required to give their answers in their own words as far as practicable)

Group-A

Answer any two questions.

1. (a) The action integral for motion under a central force is

$$S[r,\theta] = \int_0^T \left(\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)\right) dt$$

Deduce the corresponding Euler-Lagrange equations of motion. 3

(b) Show that under the infinitesimal variation $\theta(t) \rightarrow \theta(t) + \epsilon(t)\alpha$ (α being a fixed angle) of the above action there exists a conserved quantity, $mr^2\dot{\theta}$, whenever the equations of motion are satisfied. 4

(c) Show that for, $V(r) = -\frac{k}{r}$ $(k > 0)$, there exists another conserved quantity given by, $\vec{A} = \vec{p} \times \vec{L} - mk\hat{r}$, where $\vec{p}$ and $\vec{L}$ represent the linear and angular momentum respectively. 3

2 (a) Consider the system of differential equations

$$\dot{x} = (a - by)x(1 - x),$$

$$\dot{y} = -(c - dx)y(1 - y),$$

with $x > 0, y > 0$ and a, b, c, d being real constants. Introduce new variables q, p through the substitution, $x = e^q/(1 + e^q)$, $y = e^p/(1 + e^p)$, and write the corresponding system of equations. Show that the resulting system is Hamiltonian and compute the corresponding Hamiltonian. 5

(b) Consider the motion of a particle of mass m moving in one-dimension under the action of a force $F = kq$ where q denotes the position coordinate. Draw the phase portraits for both the attractive case ($k < 0$) and the repulsive case ($k > 0$) taking into consideration all possible signs of the energy E. 5

3. (a) Define a canonical transformation. Show that the following transformation is canonical $Q = \frac{m\omega q + ip}{\sqrt{2m\omega}}$, $P = i\frac{m\omega q - ip}{\sqrt{2m\omega}}$. Find the $F_1(q, Q)$ generating function of the transformation. 4

(b) How does the Hamiltonian, $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$, transform under the above canonical transformation? Obtain the equations of motion for the transformed variables Q and P and write their solutions. 3

(c) Given the Poisson brackets $\{L_i, p_j\} = \epsilon_{ijk}p_k$ and $\{L_i, L_j\} = \epsilon_{ijk}L_k$, show that $\{L_1, A_2\} = A_3$ where L_1 and A_2 represent the components of the angular momentum $\vec{L}$ and the Laplace-Runge-Lenz vector, $\vec{A} = \vec{p} \times \vec{L} - mk\hat{r}$, respectively. 3

Group-B

Answer any two questions.

5. (a) Consider the following Lagrangian

$$L = \frac{1}{2}M(\dot{\eta}_1^2 + \dot{\eta}_2^2) - \frac{K}{2}(2\eta_1^2 + 2\eta_2^2 - 2\eta_1\eta_2).$$

Find the values of the normal mode frequencies and calculate the modal matrix. Deduce the transformation that reduces the equations of motion to that of a system of linear harmonic oscillators. 5

(b) A rigid body in motion has angular velocity $\vec{\omega} = 2\hat{k}$ and its moment of inertia tensor is $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$. Calculate the angular momentum $\vec{L}$ of the rigid body. 5

6. A heavy symmetric top having mass M fixed at the bottom point is rotating with angular velocity $\vec{\omega}$. The compuonents of $\vec{\omega}$ along the body-fixed axes which coincide with the principal axes (x, y, z) of the top are:

$$\omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi, \quad \omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi, \quad \omega_3 = \dot{\phi}\cos\theta + \dot{\psi},$$

where θ, ϕ and ψ are the Euler angles describing the orientation of the top. The principal moment of inertia of the top about the symmetry (figure) axis is I_3 while $I_1 = I_2 = I_0(>> I_3)$. The centre of gravity of the top lies at a distance ℓ from the bottom most point of the top.

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M.Sc. (1st Year) 1st Semester Examination, 2021
Subject: Physics
Paper Code: Phy/CC/Th/103
Paper: PHY/Th/1S/103(For Supplementary Candidates)
(Quantum Mechanics I)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
(Use separate answer script for each group)

Group-A

Answer any two questions

1. (a) For any vector **A** show that $[\sigma, A.\sigma]=2iA\times\sigma$

   (b) The rotational part of a Hamilonian of a diatomic molecule is $\frac{1}{2I}\left(L_x^2 + L_y^2\right) + \frac{1}{I}L_z^2$, I is themoment of inertia. Find the energy eigenvalues.

   (c) Though the angular momentum components J_x, J_y, and J_z commute with J^2, we cannot have a representation in which J^2, J_x, J_y, and J_z are diagonal. Why?

   4+4+2=10

2. (a) What are stationary states? Why are they so called?

   (b) If $\mathbf{J_1}$ and $\mathbf{J_2}$ represent angular momentum operators, are $\mathbf{J_1}+\mathbf{J_2}$ and $\mathbf{J_1}-\mathbf{J_2}$ angular momenta? Explain.

   (c) Derive matrix for the operators J^2, J_z and, J_x for j=3/2

   2+4+4=10

3. (a) Obtain the matrix of Clebsh-Gordan coefficient for $j_1=1/2$ and $j_2=1$.

   (b) Find the expectation value of the potential energy in the n^{th} state of the harmonic oscillator.

   (c) For the Hamiltonian $H=a_0I+\mathbf{b}.\sigma$ where $a_0\in R$, **b** is a real vector, I is 2x2 identity matrix and σ are the Pauli Matrices, find the ground state energy.

   4+2+4=10

Sujit Pal

Group – B

Answer any two questions

4.(a)Show that the interaction picture satisfies a similar time dependent Schrodinger equation with H replaced by perturbed Hamiltonian H'.

(b) Using variational method, find the ground state energy of Helium atom. Given $<1/r_{12}> = \frac{5}{4} Z_{eff} E_H$ and E_H=-13.6 eV.

(c) Obtain the classical turning point (s) of a one dimensional harmonic oscillator.

3+5+2=10

5.(a) Calculate the 1st order correction to the ground state energy of a 1D anharmonic oscillator of mass m and angular frequency ω subject to a potential $V(x) = \frac{1}{2} m\omega^2 x^2 + bx^4$. Given the ground state wave function as

$$\psi_0^{(o)} = (m\omega/\pi\hbar)^{1/2} exp(-m\omega x^2/2\hbar).$$

(b) Obtain an expression of WKB wave function for a particle with E > V.

(c) Comments on the mass, energy,etc. of a particle moving in a potential to fulfill the condition of validity of WKB approximation in quantum mechanics.

3+5+2=10

6.(a) A particle is trapped in an infinite square well of width "b" at its bottom and its unperturbed wave function is given $\psi_n^{(o)} = \sqrt{\frac{2}{b}} \sin\left(\frac{n\pi x}{b}\right)$. If the system is perturbed by raising the floor by a constant potential V_0, find the 1st order correction to the energy in its nth state.

(b) A rigid rotator having a moment of inertia I and electric dipole moment μ executes rotational motion in a plane. Estimate the 2nd order correction to energy when the rotator is acted upon by an electric filed E in the plane of rotation. Given $E_n^0 = \frac{n^2\hbar^2}{2I}$.

(c) How do you normalize the wave function of a free particle? 3+5+2=10

Suji Pal

(a) By equating the integrals of motion corresponding to the Euler angles ϕ and ψ with $I_0 b$ and $I_0 a$ respectively, obtain an expression for the total energy where a and b are constants. Hence show that, $\dot{u}^2 = (\alpha - \beta u)(1-u^2) - (b - au)^2$, where $u = \cos\theta$, $\alpha = \frac{2}{I_0}(E - \frac{1}{2}I_3\omega_3^2)$ and $\beta = \frac{2Mg\ell}{I_0}$. 5

(b) Assuming the figure axis of the 'fast top' ($\frac{1}{2}I_3\omega_3^2 >> 2Mg\ell$) to be released at $t = 0$ and $\theta = \theta_0$ with only spinning motion of the top ($\dot{\psi} \neq 0$) find an expression for the extent of nutation. How can the effect of nutation be minimized? 5

7. (a) Show that $T^{\nu}_{\gamma\alpha} S^{\alpha\beta}_{\nu}$ is a mixed tensor of rank 2. 1.5

(b) Prove that the four-velocity vector is orthogonal to the Minkowski force. 2.5

(c) Draw the graph of energy versus momentum for (i) a particle with zero rest mass and (ii) a particle with non-zero rest mass. 2

(d) Find an expression for the Hamiltonian of a relativistic particle with rest mass m_0 and momentum p and potential energy V. Deduce the corresponding equations of motion of the particle. 4

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M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2021

Subject : Physics

PHY/CC/PR/104

Numerical Methods & Computer Applications in Physics

Time : 4 Hours Full Marks : 50

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.
Illustrate the answers wherever necessary.
(*Use separate answer scripts for each group*)

For each question, you have to send the respective codes viz. .tex and .f95 files along with the pdf outputs/screenshots of your answers to the specified email id within the stipulated time.

Group-A

Answer any one

Marks Distribution: (i) 21 [Working formula/brief theory=3, Program writing=12, Result using computer progamming/calculator using formula=3, Skill/Efficient program writing=3], (ii) 2, (iii) 2

1. (i) Write a FORTRAN program to find the value of the following integral using composite Trapezoidal rule:

$$I = \frac{1}{\sqrt{2}\pi}\int_{0.5}^{1.5} e^{-\frac{x^2}{2}}\,dx$$

(ii) How can you improve the result?
(iii) How do you find the value of $x = \sqrt{27}$? (can choose any method in numerical analysis)

2. (i) Write a FORTRAN program to find the value of the following integral using composite Simpson's 1/3 rule

$$I = \int_{0}^{\frac{\pi}{3}} \sqrt{\frac{\cos x}{2}}\,dx$$

(ii) How can you improve the result?
(iii) Between *Bisection* and *Newton-Raphson* methods of root findings, which one converges fast and why?

3. (i) Write a FORTRAN program to find a real root of the following equation using Bisection method:

$$f(x) = -10x^3 + 8x + 9 = 0$$

(ii) How can you improve the result?
(iii) Obtain ranges of all the roots. Why composite method of integration gives better result?

4. (i) Write a FORTRAN program to find a real root of the following equation using Newton-Raphson method:

$$f(x) = x^3 - x - 0.1 = 0$$

(ii) How can you improve the result?
(iii) Between *Trapezoid* and *Simpson 1/3* rules, which one gives more accurate result and why?

Group-B

Answer all the questions

5. Write down the following expressions in LaTeX

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathrm{A} + \nabla\left(\nabla\cdot\mathrm{A}\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right) = \mu_0\mathrm{J}$$

$$\partial_\alpha\partial^\alpha A^\beta = \mu_0 J^\beta$$

$$\frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}F^{\alpha\beta}\right) = \nabla_\alpha F^{\alpha\beta} = \mu_0 J^\beta$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

10

6. Write down the applications of *least square method* under physics perspective. Write a FORTRAN program to fit a straight line for the following set of data via least square fit method.

y	-3.243	-0.987	0.004	2.132	3.998
x	-3.0	-1.0	0.0	2.0	4.0

Print the values of the slope and the intercept of the fitted straight line in the output screen. Compare your numerical result with the analytical one. 2+8+5=15

Sanjib Paul