DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2020

Subject: Physics

Paper: Phy/Th/2S/201/20 (Quantum Mechanics II)

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.
Answer any two questions from each group
(Use separate answer scripts for each group)

Group-A

1. a) The operators S_x and S_y are defined as: $S_x = \frac{1}{2}\hbar\{|+><-| + |-><+|\}$ and $S_y = \frac{1}{2}\hbar\{-i|+><-| + i|-><+|\}$ where, $|\pm>$ are the eigen states of the operator S_z with eigenvalues $\pm\hbar/2$. Obtain the matrix representation of $S_{\pm}$.

   b) Obtain an expression for the conserved current corresponding to Dirac equation.

   c) A beam of spin ½ atom goes through a series of Stern-Gerlach measurements as follows:
   i) The first measurement accepts $s_z = \frac{\hbar}{2}$ atoms and rejects $s_z = -\frac{\hbar}{2}$ atoms.
   ii) The second measurement accepts $s_n = \frac{\hbar}{2}$ atoms and rejects $s_n = -\frac{\hbar}{2}$ atoms, where s_n is the eigenvalue of the operator $\vec{S}\cdot\hat{n}$, with $\hat{n}$ making an angle β in the xz- plane with respect to the z-axis.
   iii) The third measurement accepts $s_z = -\frac{\hbar}{2}$ atoms and rejects $s_z = \frac{\hbar}{2}$ atoms.
   What is the intensity of the final $s_z = -\frac{\hbar}{2}$ beam when the $s_z = \frac{\hbar}{2}$ beam surviving the first measurement is normalized to unity? How must we orient the second measuring apparatus if we are to maximize the intensity of the final $s_z = -\frac{\hbar}{2}$ beam?

   3+3+4=10

2. a) Write down the Dirac equation of an electron in an electromagnetic field. Assuming the trial solution to be $\Psi = \begin{pmatrix}\chi\\ \phi\end{pmatrix}$. Obtain the set of differential equations for χ and ϕ in the non-relativistic limit.
   b) Derive an expression for the magnetic moment operator of a Dirac particle explaining the terms present in that operator. Is there any correction needed for this magnetic moment? Comment on your answer.

   4+6=10

(b) Show that the Pauli spin matrices satisfy the following relation

$$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$$

where the terms have their usual meanings.

(c) Prove that

$$[XP_X, H] = \frac{i\hbar}{m}P_X^2 + X[P_X, V]$$

where $H = \frac{P_X^2}{2m} + V$ and V is the potential energy operator. 3+3+2=8

Group-B

*Answer question **No.5** which is compulsory and **any two** from the rest.*

5. (i) When do we apply WKB approximation for getting approximate solution of a system? Write down the condition for validity of WKB approximation. 2

(ii) A particle is trapped in an infinite square well of width a at its bottom and its unperturbed wave function is given by $\psi_n^{(0)} = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$. If the system is perturbed by raising the floor by a constant potential V_0, find the first-order correction to the energy in its n^{th} state. 2

6.(a) Prove that in Schrödinger representation of quantum mechanics any operator A_s satisfies the following relation:

$$\frac{d}{dt} < A_S >= \frac{1}{i\hbar}[A_S, H] + \frac{\partial A_S}{\partial t}.$$

(b)Using variational method, find the ground state energy of Helium atom. Given $< \frac{1}{r_{12}} >= \frac{5}{4} Z_{eff} E_H$ and $E_H = -13.6$ eV. 3 +5=8

7.(a) Calculate the first-order correction to the ground state energy of a one-dimensional anharmonic oscillator of mass m and angular frequency ω subject to the potential $V(x) = \frac{1}{2}m\omega^2 x^2 + bx^4$. Given that the ground state wave function is $\psi_0^{(0)} = (m\omega/\pi\hbar)^4 \exp(-m\omega^2/2\hbar)$.

(b) Obtain an expression of the WKB wave function for a particle with $E > V$. 3+5=8

8.(a) Obtain energy eigenvalues of a one -dimensional harmonic oscillator using the WKB approximation.

(b) Treating spin-orbit interaction $H' = W(r)\mathbf{L} \cdot \mathbf{S}$ as the perturbation, find the splitting of the ground state energy of H-atom. 3 +5=8

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DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2020

Subject: Physics

Paper: PHY/Th/2S/202/20 (Statistical Mechanics)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Answer any two questions from each group

(Use separate answer scripts for each group)

Answers to the questions should be in the candidates own words as far as practicable

Group-A

1. (a) In a quantum mechanical two state system with energy levels $E_1=0$ and $E_2=\varepsilon$, the probability of finding a particle in a state is proportional to the Boltzmann factor exp (-E/kT). Find the specific heat at constant volume C_v for the system. Draw a diagram depicting the variation of C_v with T.

(b) For a one dimensional system, show that the density of state is given by:

$$D(\varepsilon)=\frac{g}{2\pi\hbar}\sqrt{\frac{2m}{\varepsilon}}, \text{ for non-relativistic energy.}$$

$$=\frac{g}{\pi\hbar c}\frac{\varepsilon}{\sqrt{\varepsilon^2-m^2c^4}}, \text{ for relativistic energy.}$$

(5+1)+(2+2)=10

2. (a) State and prove Liouville's theorem. Discuss its physical significance.

(b) A system has two normal modes of vibration with frequencies ω_1 and $\omega_2=2\omega_1$. Show that the probability that at temperature T, the system has energy less than $4\hbar\omega_1$ is

$$x^{\frac{3}{2}}\left(1+x+2x^2\right)/Z$$

where $x=e^{-\beta\hbar\omega_1}$ and Z is the partition function. (1+4+1)+4=10

3. (a) A solid containing non-interacting paramagnetic atoms, each have a magnetic moment equal to 1 Bohr Magneton (BM), is placed in a magnetic field of 3 Tesla. Assuming that the atoms are in thermal equilibrium with the lattice. Find the temperature at which the solid must be cooled so that more than 60% of atoms are polarized with magnetic moment parallel to magnetic field. (1BM=9.27 X 10^{-24} J/T)

(b) Show that the partition function Z_N for an extreme relativistic gas consisting of N-non-interacting molecules with energy-momentum relationship $\varepsilon=cp$ is given by

$$Z_N=\frac{1}{N}\left[8\Pi V\left(\frac{kT}{hc}\right)^3\right]^N.$$

Hence deduce the pressure of the gas.

(c) Under what conditions the postulates of equal a-prori probability are applicable?

4+5+1=10

Group-B

4. (a) The single particle energy of a Bose gas with mass m, volume V and spin $S = 0$ is given by

$$\varepsilon = \frac{p^2}{2m} + \Lambda\Delta,$$

where Δ is a constant positive number and the kinetic part of the particle has been denoted as $\frac{p^2}{2m}$. Determine the critical temperature for the Bose-Einstein condensation. Also state what happens to that temperature for $\Delta \gg k_B T$. The symbols carry their usual meaning and Λ is an integer which possesses the values $\Lambda = 0, 1$.

(b) Consider an electron is in an external magnetic field viz. $\mathbf{B} = \eta \hat{e}_x$, where η is a constant and $\hat{e}_x$ is the unit vector along x. If the Hamiltonian of the system is expressed as $\hat{H} = -\mu_B(\hat{\sigma} \cdot \mathbf{B})$, then find the density matrix of the system by considering it to be a canonical one. Thereafter also find $< \sigma_x >$. The symbols carry their usual meanings. 5+5=10

5. (a) Let us take an atomic nucleus of Helium consists of a gas of 0.18 nucleons in a volume of 1 fm^3. There are two kinds of nucleons viz. protons and neutrons with spins $S = 1/2$ and masses $m_n \approx m_p = 1$ GeV. Evaluate the Fermi energy for the system and make a comment on the nature of the system at ambient temperature. You can assume that the constituents are independent with a single particle energy (ϵ) approximation as $\epsilon = p^2/2m_n$, where p is the absolute value of the momentum. Useful data: $h = 7 \times 10^{-24}$ GeVs, $k_B = 8.61 \times 10^{-5}$ eV/K, $m_e = 0.5$ MeV and 10^{-14} cm $\approx 7 \times 10^{-25}$ s (in the scale of the present problem). The symbols carry their usual meanings.

(b) Draw a neat graph for C_v vs T for an ideal Bose gas and discuss about the various physical phenomenon happens inside the system for temperature $T < T_0$, $T = T_0$ and $T > T_0$, where T_0 is the condensation temperature for the system. 5+5=10

6. (a) Implementing the Saha ionization formula, prove the following relation for a pure and partially ionized hydrogen gas star:

$$\left[\frac{\partial x}{\partial(\ln \rho)}\right]_T = -\frac{x^2 - x}{x - 2},$$

where the symbols carry their usual meaning with x being the degree of the ionization of the system.

(b) Lets assume there are N noninteracting and distinguishable particles in a quantum mechanical system. The system is a two level one. Each one of the particle can be present either in state 0 with energy $E_0 = \lambda$ or in the state 1 with energy $E_1 = \Lambda$. A grad student somehow calculated that the number of particles present in state 0 is n. Calculate the approximated value of the entropy of the system in terms of internal energy (U) of the system. 5+5=10 o/e

Diamond Harbour Women's University

M.Sc 1st Year 2nd Semester Examination 2020

Subject: Physics

Paper: PHY/Th/2S/203 (General Electronics)

Time: 2 Hours | Full Marks: 40

(The figures in the margin indicate full marks.)

Group-A

Answer any two questions from this group

1. a) If the lower and the upper half -power frequencies of an RC-coupled amplifier are 30 Hz and 300 KHz, respectively, find the gain relative to the mid-frequency gain at 60 Hz and 600 KHz.

b) Obtain the expression for voltage gain of an RC-coupled amplifier in the mid frequency range. Discuss the nature of the voltage gain in low and high frequency range.

c) How are MOSFETs different from JFETs. How is channel formed in a MOSFET? Distinguish between n and P channel of a MOSFET. 3+ (3+1)+3=10

2.a) Explain with the help of a block diagram the working principle of a negative feedback amplifier and find its voltage gain.

b) A transistor in CE class A amplifier circuit has $h_{fe} = 100$ and $h_{oe} = 30 * 10^{-6} S$. If the load resistance is 3 KΩ, find the current gain.

c) Derive an expression for the width of the depletion region across a p-n junction, in terms of impurity concentrations. 2+3+5=10

3. a) Explain built-in potential with the help of the energy band diagram and derive the expression for built-in potential.

b) Show that the diffusion capacitance of a step-graded p-n junction is given by

$C_T = \varepsilon A\sqrt{\dfrac{eN_d}{2\varepsilon(V_B+|V_A|)}}$ Where N_d is the donor ion concentrations, V_B is the barrier potential and V_A is the applied voltage. (2+1+3)+4=10

Gbnae

Group - B

Answer any two questions from this group

4. (a) Using op-amp, deduce an expression of the output voltage of a non-inverting adder.

   (b) Draw the circuit using op-amp to obtain the output V_o (V_1, V_2 - inputs): $V_o = 5V_1 + 2V_2$.

   (c) "In an op-amp, a small input voltage produces very high output." -- How is the overall voltage gain controlled in an op-amp? 4+4+2=10

5. (a) Draw the circuit diagram of a logarithmic amplifier using op-amp along with a n-p-n transistor.

   (b) Draw an analog circuit using op-amp to solve the following simultaneous equations:

   $$x + 10\,y + 3 = 0 \quad \text{and} \quad 2\,x + 5\,y + 5 = 0.$$

   (c) Is there any dependence of the overall voltage gain of an ideal op-amp on the frequency of the input signal? If so, mention that. 4+4+2=10

6. (a) Design a J-K flip-flop and write down its truth table.

   (b) Draw the circuit diagram using op-amp to obtain the out voltage $V_o = -2V_i$.

   (c) What is the reason of using negative feedback in an op-amp? 4+4+2=10

Diamond Harbour Women's University
M.Sc 1st year 2nd Semester Examination 2020
Subject: Physics
Paper: PHY/Th/2S/204 (Electrodynamics)

Time : 2 hours *Full Marks: 40.*

Answer any 4 questions taking two from each group.
Answers to the questions should be written in the candidates own words as far as practicable.

GROUP A

1.(a) Explain what is meant by gauge freedom. Derive the equations satisfied by the scalar and vector potentials in the Lorentz gauge.

(b) Evaluate the expression

$$\phi(\mathbf{x}, t) = \int \frac{\|f(\mathbf{x}', t')\|}{|\mathbf{x} - \mathbf{x}'|} d^3x' \quad \text{for} \quad \|f(\mathbf{x}', t')\| = \delta(x')\delta(y')\delta\left(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}\right).$$

Note that

$$\delta(f(z)) = \sum_i \frac{\delta(z - z_i)}{|\left(\frac{df}{dz}\right)_{z=z_i}|}$$

where z_i's denote the zeros of the function $f(z)$. [5+5=10]

2.(a) Use the following expression for the vector potential $A(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}', t_r)}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'$, where t_r denotes the retarded time, to obtain an explicit expression for the magnetic field $\mathbf{B}(\mathbf{x}, t)$. Comment on your result.
(b) Show that the scalar Lienard-Wiechert potential for an proton moving with uniform velocity $\mathbf{v}$ may be expressed as

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{e}{s\sqrt{1 - \beta^2 \sin^2\theta}}, \quad \beta = \frac{\mathbf{v}}{c}$$

where $\mathbf{s} := \mathbf{r} - \mathbf{v}t$ and θ is the angle between $\mathbf{s}$ and $\mathbf{v}$. [4+6=10]

3. (a) An insulating circular ring of radius R lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \cos\phi$, where λ_0 is a constant and ϕ denotes the azimuthal angle. (i) Calculate the dipole moment of the circular ring. (ii) If the ring is now set spinning at a constant angular velocity ω about the z axis determine the power radiated.

(b) Obtain an expression for the Thompson scattering cross-section expressing your answer in terms of the the classical electron radius. [6+4=10]

GROUP B

4. Given the following expression for the power radiated per unit area per unit time-at-particle per unit solid angle,

$$\frac{dP'}{d\Omega_s} = \kappa \frac{R^2}{\mu_0 c} \left\|\left| \frac{1}{c\kappa^3 R} \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \frac{\mathbf{v}}{c}) \times \frac{\dot{\mathbf{v}}}{c} \right] \right|\right\|^2 \left(\frac{q}{4\pi\epsilon_0} \right)^2,$$

show that when the acceleration is parallel to the velocity

$$\frac{dP'}{d\Omega_s} = \frac{q^2 \dot{v}^2}{(4\pi)^2 c^3 \epsilon_0} \left[\frac{\sin^2\theta}{(1-\beta\cos\theta)^5} \right],$$

where θ is the angle between $\hat{\mathbf{R}}$ and $\mathbf{v}$ and $\|\cdots\|$ means that the contents are to be evaluated at the retarded time. Note that $\mathbf{R} = (\mathbf{r} - \mathbf{r}')$ and $\kappa = \left(1 - \hat{\mathbf{R}} \cdot \frac{\mathbf{v}}{c}\right)$. Hence find in case of ultra relativistic speeds (i.e., when $\beta = v/c \longrightarrow 1$) the direction in which maximum radiation is emitted expressing your answer in terms of $\gamma = (1-\beta^2)^{-1/2}$.

[6+4=10]

5.(a) Consider the following action, $S = \int (j^\mu A_\mu + \alpha F_{\mu\nu} F^{\mu\nu}) d^4x$ with A_μ and j^μ being the 4-potential and 4-current density respectively and $F_{\mu\nu}$ is the anti-symmetric electromagnetic field tensor. Use the variational principle to derive the corresponding equation for the field tensor.

(b) There is only a uniform electric field $\mathbf{E}$ in an inertial frame S. Find the magnitudes and directions of $\mathbf{E}'$ and $\mathbf{B}'$ in an inertial system S' moving relative to S at a constant relativistic velocity $\mathbf{v}$ at an angle α to $\mathbf{E}$.

[5+5=10]

6. Given the following transformation laws for the electric and magnetic fields under a Lorentz boost:

$$\mathbf{E}'_{\|} = \mathbf{E}_{\|}, \quad \mathbf{E}'_{\perp} = \gamma(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}_{\perp}), \quad \mathbf{B}'_{\|} = \mathbf{B}_{\|}, \quad \mathbf{B}'_{\perp} = \gamma(\mathbf{B}_{\perp} - \frac{\vec{\beta}}{c} \times \mathbf{E}_{\perp})$$

(a) Show that (i) if $\mathbf{B} = 0$ and $\mathbf{E} \neq 0$ in an inertial frame S then in the inertial frame S', $\mathbf{B}' = -(\mathbf{v} \times \mathbf{E}')/c^2$ and (ii) if $\mathbf{E} = 0$ and $\mathbf{B} \neq 0$ in S then in S', $\mathbf{E}' = (\mathbf{v} \times \mathbf{B}')$.

(b) An infinite straight wire with charge λ per unit length lies stationary along the x-axis of an inertial frame S. Obtain the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ in an inertial system S' which moves at a constant relativistic velocity $\mathbf{v}$ along the common $x - x'$ axis and prove that $\mathbf{E} \cdot \mathbf{B} = 0$.

[3+2+5=10]