DHWU

M.Sc. (1^{st} Year) 1^{st} SEMESTER EXAMINATION 2020

Subject : Physics

Paper: Phy/Th/1S/101

Mathematical Methods

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks

(***Use separate answer scripts for each group***)

Group-A

Answer question no. 1 which is compulsory and any two from the rest.

1.(i) Show that every linear ordinary differential equation of the form, $y'' + p(x)y' + q(x)y = 0$, may be reduced to the standard form, $v'' + \omega(x)v = 0$, by the transformation $y = v(x)e^{-\int \frac{p(x)}{2}dx}$ where $\omega(x) = q(x) - \frac{1}{4}p^2(x) - \frac{1}{2}p'(x)$. 2

(ii) Prove that if $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\{f(at)\} = \frac{1}{a}F(s/a)$. Here $\mathcal{L}$ represents the Laplace transformation of the argument with respect to the variable s. 2

2.(a) Use the method of variation of parameters to obtain the general solution of the differential equation $y'' + y = cosec\ x$. 4

(b) Prove that a second-order Fuchsian ordinary differential equation with two regular singular points may be reduced to the Euler equation by an appropriate transformation. 4

3.(a) Solve the ordinary differential equation, $ty'' + (1-2t)y' - 2y = 0$, with initial conditions $y(0) = 1$ and $y'(0) = 2$ using Laplace transform. 4

(b) Consider the Euler equation $z^2w''(z) + 4zw'(z) + 2w(z) = 0$. Show that $z = \infty$ is a regular singular point of this equation with exponents 2 and 1 respectively. 4

4.(a) Show that $V = \{(x,y,z)\big| x,y,z \in \mathbb{R},\quad x + y = 11\}$ is not a subspace of $\mathbb{R}^3$. 2

(b) Consider the vector $x = (4,5)$ in $\mathbb{R}^2$ with respect to the standard basis. Express the vector x with respect to the basis $A = \{(1,1),(-1,2)\}$ 3

(c) Calculate the best approximate solution of the following inconsistent system: $C + D = 1$, $C + D = 2$ and $C + 3D = 2$ using the least square method. 3

Group-B

Answer question No.5 which is compulsory and any two from the rest.

5.(i) If the imaginary part of an analytic function is $2x(1-y)$, determine the real part and find $f(z)$. 2

(ii) Show that the eigenvalues of a Sturm-Liouville system are real. 2

6.(a) The generating function of the Hermite polynomials is $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$. Show that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$. 3

(b) Expand the function $f(z) = 1/(1+z^2)$ in a Laurent series for $|z| > 1$. 2

(c) Show that if $f(z)$ is analytic everywhere inside and on a simple closed contour C except at $z = a$ which is a pole of order n then the residue of $f(z)$ at $z = a$ is given by

$$\lim_{z \to a} \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} \{(z-a)^n f(z)\}.$$

3

7.(a) Evaluate

$$\oint_C \frac{e^z dz}{(z-1)(z+3)^2}$$

where C is given by the circle $|z| = 3/2$. 3

(b) If $|f(z)| \leq \frac{M}{R^k}$ for $z = Re^{i\theta}$, where $k > 0$ and M are constants prove that,

$$\lim_{R \to \infty} \int_\Gamma e^{imz} f(z) dz = 0,$$

where Γ is the semi-circular arc in the upper half plane and m is a positive constant. 5

8.(a) Evaluate the inverse Laplace transform $\mathcal{L}^{-1}\{\frac{1}{s-2}\}$ by using the complex inversion formula. 4

(b) Construct the Green's function for the boundary value problem, $y'' + \omega^2 y = f(x)$, where $f(x)$ is a known function and y satisfies the boundary conditions $y(0) = 0$ and $y(L) = 0$. 4

DHWU

M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2020

Subject : Physics

Paper: Phy/Th/1S/102

Classical Mechanics

Time : 2 Hours

Full Marks : 40

The figures in the margin indicate full marks

(Use separate answer scripts for each group)

Group-A

Answer question No. 1 which is compulsory and any two from the rest.

1.(i) Determine the extremals of the following functional assuming fixed values of $q(t_0)$ and $q(t_1)$ 2

$$J[q] = \int_{t_0}^{t_1} (q^2 + \dot{q}^2 - 2qt)dt$$

(ii) For the Hamiltonian $H = p^2/2 - 1/(2q^2)$ show that $I(q, p, t) = pq/2 - Ht$ is a constant of motion. 2

2.(a) A bead of mass m can slide without friction on a straight wire which is constrained to move in the (x_1, x_2) plane around the origin with constant angular velocity ω. Write down the constraint equation in Cartesian coordinates and deduce an expression for the force of constraint by using the Lagrange undetermined multiplier method. Obtain the equations of motion. 5

(b) Consider the transformation

$$P = -q - \sqrt{p + q^2}, \quad Q = -q^2 - aq\sqrt{p + q^2}$$

Determine for which values of the real parameter a the transformation is completely canonical. Compute the generating function $F(q, P)$. 3

3. (a) Two blocks having mass $m_1 = 2m$ and $m_2 = m$ are connected to two rigid supports by massless springs of force constants $k_1 = 4k, k_2 = k$ and $k_3 = 2k$ respectively. All motion is horizontal and all springs are unstretched when the blocks are at rest. (See Fig. 1) (i) Choose as generalized coordinates the displacement of each block from its equilibrium position, and write the Lagrangian.(ii) Compute the frequencies of small oscillations. (2+3)

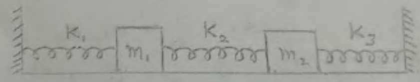

Figure 1: Question no. 3.(a)

(b) The equation of motion of a damped linear harmonic oscillator is given by

$$\ddot{q} + \alpha\dot{q} + \omega^2 q = 0, \quad \alpha > 0$$

Derive a Lagrangian for this equation by deducing a Jacobi Last Multiplier. 3

4.The Lagrangian for the motion of a charged particle in a magnetic field is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - bx\dot{y}.$$

(i) Show that the Lagrangian is non-singular by computing the Hessian matrix. (ii) Write down the corresponding Hamiltonian and obtain the canonical equations of motion.(iii) Set up the Hamilton-Jacobi equation for the Hamiltonian. (iv) Use separation of variables to deduce a complete integral of the Hamilton-Jacobi equation. 1+2+1+4=8

Group-B

*Answer question **No.5** which is compulsory and **any two** from the rest.*

5. (i) Find the axis of rotation corresponding to the following rotation matrix

$$\frac{1}{9}\begin{pmatrix} -7 & 4 & 4 \\ 4 & -1 & 8 \\ 4 & 8 & -1 \end{pmatrix}$$ 2

(ii) What is the speed of a γ-ray source of energy 14.4 KeV w.r.t an obbserver, which measures γ-energy to be 14.401 KeV? 2

6.(a) A rod having negligible mass is connected to a disc having mass M and radius R through its center. The whole system rotates about the rod keeping its other end fixed on the ground. Assuming torque-free motion write down the Euler equations of motion and show that the body cone rolls without slipping inside the space cone. 4

(b) The inertia tensor of a solid cube having mass M and each side a about a set of axes coinciding with one of its corners is given by

$$\frac{Ma^2}{12}\begin{pmatrix} 8 & -4 & -4 \\ -4 & 8 & -4 \\ -4 & -4 & 8 \end{pmatrix}$$

If the cube rotates about its diagonal passing through the origin with uniform angular velocity ω, find its angular momentum and rotational kinetic energy. 4

7.(a) Obtain the two integrals of motion of a heavy symmertrical top under gravity using Euler's equations expressing the componnents of angular velocity in terms of Euler angles. 5

(b)Define Minkowski force and 4-velocity vector. Show that they are orthogonal to each other. 3

8.(a) Show that $A^\mu B^\nu_\mu$ is a contravariant vector. 2

(b) Show that the rest mass of the product nucleus is greater than the sum of rest masses of the colliding nuclei for a perfectly elastic collision. 3

(c) A rod of proper length $1m$ is moving with a speed of $0.6c$ in $+x$ direction as seen in the frame S. In the same frame a particle is found to move in $-x$ direction with a speed of $0.8c$. Find the time taken for the particle to cross the rod in (i) rod frame S' 3

DHWU

M.Sc. (1^{st} Year) 1^{st} SEMESTER EXAMINATION 2020

Subject : Physics

Paper: Phy/Th/1S/103

Quantum Mechanics-I

Time : 2 Hours | Full Marks : 40

The figures in the margin indicate full marks

(Use separate answer scripts for each group)

Group-A

Answer question No. 1 which is compulsory and any two from the rest.

1. (i) For any vector **A** show that $[\sigma, \mathbf{A}\cdot\sigma] = 2i\mathbf{A}\times\sigma$. 2

(ii) Show that commuting operators have a common set of eigenfunctions. 2

2.(a) For a one dimensional harmonic oscillator, use creation and annihilation operators to, show that

$$(\Delta x).(\Delta p_x) = \left(n + \frac{1}{2}\right) i\hbar.$$

(b) Find the expectation value of the potential energy in the n^{th} state of the harmonic oscillator.

(c) Derive matrix representations for the operators J^2, J_z, J_x and J_y for $j = 1/2$. 3 +2+3=8

3.(a) For the eigenstates of J^2 and J_z show that

$$< J_x >=< J_y >= 0$$

$$< J_x^2 >=< J_y^2 >= \frac{1}{2}\hbar^2[j(j+1) - m^2].$$

(b) How many angular momentum states arise for a system with two angular momenta $j_1 = 1$ and $j_2 = 1/2$? Specify the states. (2+3)+3=8

4.(a) If the eigenvalues of J^2 and J_z are given by $J^2|\lambda, m >= \lambda|\lambda, m >$ and $J_z|\lambda, m >= m|\lambda, m >$ then show that $\lambda \geq m^2$.

uli spin matrices satisfy the following relation

$$\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$$

heir usual meanings.

$$[XP_X, H] = \frac{i\hbar}{m}P_X^2 + X[P_X, V]$$

nd V is the potential energy operator. 3+3+2=8

Group-B

on No.5 which is compulsory and any two from the rest.

WKB approximation for getting approximate solution of a system?
n for validity of WKB approximation. 2

d in an infinite square well of width a at its bottom and its unperturbed
by $\psi_n^{(0)} = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$. If the system is perturbed by raising the floor
l V_0, find the first-order correction to the energy in its n^{th} state. 2

hrodinger representation of quantum mechanics any operator A_s satisfies

$$\frac{d}{dt} < A_S >= \frac{1}{i\hbar}[A_S, H] + \frac{\partial A_S}{\partial t}.$$

hod, find the ground state energy of Helium atom. Given $< \frac{1}{r_{12}} >=$
6 eV.
3 +5=8

order correction to the ground state energy of a one-dimensional
or of mass m and angular frequency ω subject to the potential $V(x) =$
that the ground state wave function is $\psi_0^{(0)} = (m\omega/\pi\hbar)^{\frac{1}{4}}\exp(-m\omega x^2/2\hbar)$.

of the WKB wave function for a particle with E V. 3+5=8

nvalues of a one -dimensional harmonic oscillator using the WKB

nteraction $H' = W(r)\mathbf{L}\cdot\mathbf{S}$ as the perturbation, find the splitting
y of H-atom.
3 +5=8