DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2019

Subject : Physics

Paper : Phy/Th/2S/201

(Quantum Mechanics II)

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks

(Use separate answer scripts for each group)

Group-A

(Answer question 1 and any two from the rest)

1. a) A quantum mechanical state of a particle, with Cartesian coordinates x, y and z is described by the normalized wave function.

$$\psi(x,y,z) = \frac{\alpha^{5/2}}{\sqrt{\pi}}\, z \exp\left[-\alpha(x^2+y^2+z^2)^{1/2}\right].$$

Show that the system is in a state of definite angular momentum and give the value of L^2 associated with the state. 2

b) Show that the Hamiltonian of Dirac equation – H for free particle commutes with the operator $\vec{\sigma}\,.\vec{P}$ where $\vec{P}$ is the momentum operator and $\vec{\sigma}$ is the Pauli spin operator in the space of four component spinors. 2

2. a) Show that $\hat{\vec{p}}$ is a vector operator.

b) Obtain 2 x 2 matrix representation of the rotation operator, $D(\hat{n}, \varphi)$ about an arbitary direction $\hat{n}$ through an angle φ in Pauli's two-component formalism. Assuming $\varphi = \frac{\pi}{2}$ and $\hat{n} = \hat{z}$; find an expression for $D(\hat{n}, \varphi)$. What will be the rotated state when the above rotation operator acts on the state $|\alpha\rangle = \begin{pmatrix}1\\0\end{pmatrix}$?

2+(3+2+1)=8

3. a) Show that $\vec{L}\cdot\vec{S}$ is an ordinary scalar and $\vec{L}\cdot\vec{x}$ is a pseudoscalar.

b) A system comprising of three equal mass non-interacting distinguishable spin ½ particles are in a box of length L. Calculate the three lowest energy levels together with their degeneracies.

c) Recover Klein-Gordon equation from the Dirac equation. (1.5+1.5)+3+2=8

4. a) Write down the Dirac equation in momentum space for spin ½ particles assuming 4-component spinors denoted by $u(p)$. Hence show that, $\bar{u}^{(s)}u^{(s)} = 2m$, where s stands for spin and m denotes mass of the particle.

P.T.O.

b) Consider the Dirac equation in one dimension :

$$H\psi = i\frac{\partial\psi}{\partial t}$$, where $H = \alpha p_z + \beta m + V(z)$,

$\alpha = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, I being 2 x 2 unit matrix.

i) Show that $\sigma = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$ commutes with H.

ii) Use the result of (ii) to show that the one dimensional Dirac equation can be written as two coupled first order differential equations. (1+3)+(2+2)=8

Group-A

(Answer question 5 and any two from the rest)

5. a) Why is the Ramsauer-Townsend effect in connection with scattering problem observed for noble gas atoms ?

   b) When an atom or ion is interacts with electromagnetic radiation of visible region, why dipole approximation come into play ?

6. a) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, for a constant perturbation : $H'(t') = H'$ for $0 \leq t' \leq t$; zero otherwise, obtain Fermi Golden rule for the transition rate from a discrete state i to a continuum state k.

   b) Using standard expression of scattering amplitude $f(\theta, \phi)$ due to 1st Born approximation, find an expression of the differential scattering cross section for a shielded Coulomb potential of the forn $V(r) = \frac{Z_1 Z_2 e^2 e^{-r/l}}{r}$, l being the shielding distance. 4+4=8

7. a) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, for a harmonic perturbation : $H'(\mathbf{r},t') = 2H'(\mathbf{r})\cos(\omega t')$, obtain transition probability from a discrete state i to a discrete state k for stimulated emission and absorption.

   b) Find the total scattering cross section of a particle, in the low energy limit, by a square well potential of the form. $V(r) = -V_0$, for $0 < r < a$
   $= 0$, for $r \geq a$, a= width of the potential well.

8. a) In case of scattering of a particle by a spherically potential $V(r)$, show that in the asymptotic region the scattered wave function can be taken of the following form (terms having their usual meanings). 4+4=8

$$\Psi(\mathbf{r})_{scat} = f(\theta, \phi)\frac{e^{ikr}}{r}$$

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M.Sc. (1st Year) 2nd Semester Examination, 2019

Subject : Physics

Paper : Phy/Th/2S/202

(Statistical Mechanics)

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks

Group-A

First question is Compulsory and answer any two from the remaining three questions

1. a) A free particle of mass m has position x and momentum p and it is confined in a one dimensional box (o, ℓ). Suppose its energy lies between E and E+dE. Draw the classical phase-space indicating the region of space which is accessible to the particle.

   b) Find the partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε. 2+2=4

2. a) For a three dimensional system of free electrons, show that the density of states is given by

$$D(\varepsilon) = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \quad \text{for non-relativistic energy}$$

$$= \frac{g}{2\pi^2}\frac{E\sqrt{E^2 - m^2c^4}}{(\hbar c)^3} \quad \text{for relativistic energy.}$$

   b) In a quantum mechanical two state system with energy levels $E_1=0$ and $E_2=\varepsilon$, the probability of finding a particle in a state is proportional to the Boltzmann factor exp (-E/kT). Find the specific heat at constant volume C_v for the system. Draw a diagram depicting the variation of C_v with T. (2+2)+4=8

3. a) Determine the phase trajectory of a bullet of unit mass fired straight upwards with an initial speed of 392m/s. Acceleration due to gravity is approximately 10 m/s^2.

   b) Prove that for a system in a grand canonical ensemble.

$$\langle E \rangle = \left(\frac{\mu}{\beta}\frac{\partial}{\partial\mu} - \frac{\partial}{\partial\beta}\right)\ln Z$$

4+4=8

4. a) A system with just two energy levels is in thermal equilibrium with a heat reservoir at a temperature 600K. The energy gap between the levels is 0.1 eV. Find (i) the probability that the system is in higher energy level and (ii) the temperature at which the probability will equal 0.25.

P.T.O.

b) The Hamiltonian of a paramagenetic substance is $H = -\sum_{i=1}^{N} \vec{\mu}_i . \vec{H}$.

Show that for such a substance at high temperature the magnetic susceptibility is

$$\chi_T = \frac{N\mu^2}{3kT}$$

(2+2)+4=8

Group-B

First question is Compulsory and answer any two from the remaining three questions

5. a) Consider a system of N non-interacting particles ($N \geq 1$) in which the energy of each particle can have two values i.e. either 0 or E (> 0). If the total energy of the system is U *and* n_0 and n_1 are the occupation numbers of the energy levels 0 and E respectively then find the entropy of the system.

b) If the partition function of an ensemble with two quantum states is $z = 2e^{\beta\epsilon/2}\cosh(\beta\epsilon/2)$ then find the corrosponding energy eigenvalues. 2+2=4

6. a) Consider an electron in an external magnetic field viz. $\vec{B} = B_0\hat{e}_z$, where B_0 is constant and $\hat{e}_z$ is the unit vector along z. If the Hamiltonian of the system is $\hat{H} = -\mu_B(\hat{\sigma} \cdot \vec{B})$, then find its density matrix in canonical ensemble. Thereafter, also find $< \sigma_z >$. The symbols carry their usual meanings.

b) Our universe is filled with black body radiation (photons) at a temperature 3.K. This is thought to be one of the basic phenomenon of the early developments of the universe following the big bang. Express the photon number density in terms of temperature and other universal constants. (Note : A certain numerical cofactor may be left in form of a dimensionless integral in the calculation.) (3+2)+3=8

7. a) Draw graph for C_v vs T for an ideal Bose gas and discuss the various physical phenomena happens inside the system for temperature $T < T_0$, $T = T_0$ *and* $T > T_0$, where T_0 is the condensation temperature for the system.

b) Consider a system of N non-interacting particles at temperature T. Each particle possesses a vibrational energy $\varepsilon = (n + 1/2)\hbar\omega_0$ where n = 0, 1, 2, 3...... and ω_0 is the natural frequency of vibration of each particle. Find the partition function of this system. (2+2+2)+2=8

8. What do you mean by a white dwarf ? Estimate the total energy and the pressure of a white dwarf by implementing the Femi-Dirac statistics for degenerate electorns. 2+6=8

4=8

DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2019

Subject : Physics

Paper : Phy/Th/2S/203

(General Eletronics)

Time : 2 Hours

Full Marks : 40

The figures in the margin indicate full marks

(Use separate answer scripts for each group)

Group-A

Answer question 1 and any two from the rest

1. a) Draw the pole-zero plot of the following transfer function 2

$$G(s) = \frac{(s+1)(s-2)}{(S+3)(s+4)(s^2+4)(s+2-3j)(s+2+3j)}$$

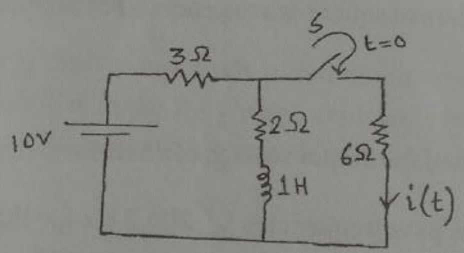
b) State the effect of negative feedback in an amplifier. 2

2. a) What is a TRIAC ? Explain its working principle with the characteristic curve.

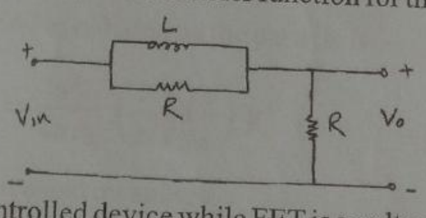
b) Obtain an expression for depletion layer width in the context of Depletion Layer Model.

(1+3)+4=8

3. a) The circuit in the fig. was initially in a state with the switch 'S' is kept open for a long time and then closed at t=0. Find i(t) at $t > o$ for the following network using Laplace transform.



b) Define transfer function. Find the transfer function for the following network.



4+(1+3)=8

4. a) A BJT is a current controlled device while FET is a voltage -controlled device, Justify.

b) An n channel JFET gives a saturation voltage V_{Dsat}= 4V for V_{GS} = -1 V. Another n channel JFET gives V_{Dsat}=6.5 V for the same V_{GS}. Compare the pinch off voltage of the two JEETs. The symbols have their usual meaning.

P.T.O.

c) The open loop gain of an amplifier changes 20 percent due to change in the parameters of the active amplifying device. If a change of gain of 2% is allowable, what type of feedback has to be applied ? If the amplifier gain with feedback is 10, find the minimum value of the feedback ratio and the open loop gain. 2+3+3=8

Group-B

Answer question 5 and any two from the rest

5. i) Obtain OR gate using 4 : 1 MUX 2

ii) The instantaneous voltage of a frequency modulated wave is 2

$$e=0.25\,\text{Sin}\,(4.2\times10^{7}t+0.1\,\text{Sin}\,10^{5}t).$$

Find maximum swing of the frequency of the modulated wave from the carrier wave.

6. i) For amplitude modulation of a low frequency audio waves, obtain an expression of the modulated wave and show that the band width is independent of the carrier wave.

ii) Explain the working principle alongwith circuit of an Encoder. 5+3=8

7. i) Design a first order active high-pass filter at a cut-off frequency 0.2 kHz with a pass band gain of 2. Necessary formulas are to be deduced.

ii) Draw the circuit diagram of square-wave generator of oscillation frequency f_o=1 KHz.

5+3=8

8. i) Deduce an expression of the output voltage of a non-inverting adder using Op-Amp.

ii) A band-pass filter has pass frequencies f_L = 200.2 Hz f_H=1kHz and a pass-band gain 4. What is the total voltage gain at a frequency of 400 Hz. What type of pass-filter is it ?

4+4=8

DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2019

Subject : Physics

Paper : Phy/Th/2S/204

(Electrodynamics)

Time : 2 Hours **Full Marks : 40**

Group-A

Question No. 1 is compulsory

1. a) Prove the following identity (Green's theorem)

$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)d^3r = \oint_S \left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right]da$, where ϕ and ψ are arbitrary scalar functions. 2

b) An infinitesimal thin uniformly charged rod of length L is rotated around the axis perpendicular to it going through its end with angular frequency $\omega \le c/L$.. The total charge on the rod is Q. Find the electric dipole moment of the rotating rod. 2

Answer any two of the following :

2. i) An infinite straight wire placed along the z axis carries a current $I(t) = k\theta(t)$ where k is a constant and, $\theta(t) = 1(t > 0)$ and zero otherwise. Find the vector potential at a perpendicular distance s from the wire.

ii) A particle of charge q moves in a circle of radius a at constant angular velocity ω Assume that the circle lies in the x - y plane centered at the origin. At time t = 0 the charge is at $(a, 0)$, on the positive x - axis. Find the Lienard-Wiechert potentials for points on the z - axis.

3+5=8

3. For radiation from relativistic particles show that when the velocity υ is parallel to the acceleration $\dot{\upsilon}$, the total power radiated by the particle is

$$P' = \frac{q^2}{4\pi\epsilon_0}\left(\frac{2}{3c}\right)\left(\frac{\dot{v}}{c}\right)\gamma^6$$

where the symbols have their usual meanings.

4. Consider a potential problem in the half-space defined by z ≥ 0, with Dirichlet boundary conditions on the plane z = 0 (and at infinity).

i) Write down the appropriate Green's function $G(x, x')$.

P.T.O.

ii) If the potential on the plane $z=0$ is specified to be $\Phi = V$ inside a circle of radius 'a' centered at the origin, and $\Phi = 0$ outside that circle, find the general expression for the potential at a point P specified in terms of cylindrical coordinates (ρ, ϕ, z).

iii) Show that, along the axis of the circle ($\rho = 0$), the potential is given by 2+4+2=8

$$\Phi = V\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)$$

Group-B

Question No. 5 is compulsory

5. i) In terms of the field tensor the inhomogeneous Maxwell's equation is

$$\partial_\nu F^{\mu\nu} = -\frac{1}{\epsilon_0 c} J^\mu$$

Show that this is consistent with the equation of continuity. 2

ii) show that $\mathbf{E} \cdot \mathbf{B}$ is an invariant. 2

(Answer any two of the following)

6. Considering radiation from an orbitary source show that the electric and magnetic fields in the radiation zone may be approximated by

$$\mathbf{E}^{rad}(\mathbf{r}, t) \approx \frac{\mu_0}{4\pi r}[\hat{r} \times (\hat{r} \times \ddot{\mathbf{p}}(t_0))]$$

$$\mathbf{B}^{rad}(\mathbf{r}, t) \approx \frac{\mu_0}{4\pi r c}\hat{r} \times \ddot{\mathbf{p}}(t_0)$$

where $t_0 = t - r/c$ and $\mathbf{p}(t_0)$ represents the dipole moment of the charge distribution. Hence show that the total power radiated is given by 6+2=8

$$P = \frac{\mu_0}{6\pi c}\ddot{p}^2$$

7. A point charge q is placed at the origin of an inertial frame S' which moves with constant velocity υ with respect to an inertial frame S along the common x – axis. The origins of S and S' are assumed to coincide at $t = t' = 0$. An observer in the S frame is situated such that at closest distance of the moving charge from the observer is equal to b. Find the electric and magnetic fields measured by the observer in the S frame. Sketch the transverse electric field. Find the distance over which the transverse electric field remains appreciable. 6+2=8

8. a) Consider the following action

$$S = \int_a^b -mc\,ds - qA_\mu dx^\mu$$

by varying this action obtain the corresponding equation of motion.

b) If $\mathbf{E} \cdot \mathbf{B} = 0$ but $|\mathbf{E}| \neq |c\mathbf{B}|$, then find the inertial frame in which the field is purely electric. 6+2=8