DHWU

M. Sc. (1ST Year) 1ST SEMESTER EXAMINATION 2019

Subject : Physics

Paper : Phy/Th/1S/101

Mathematical Methods

Time : 2 Hours **Full Marks : 40**

Group – A

(*Answer Question No. 1 which is compulsory and **any two** of the rest*)

1. a) Are the functions e^x, xe^x, x^2e^x linearly dependent or independent? 2

   b) Show that the vectors $(2,-1,-2)', (1,0,1)'$ and $(1,4,1)'$ are mutually orthogonal. 2

2. a) Construct a linearly independent set of vectors from $W = \{(2,1,1)', (-1,-1,1)', (1,2,3)', (3,1,3)', (0,-1,-3)'\}$.

   b) Find the eigenvalues and the bases for the eigenspaces of the matrix.

   $A = \begin{pmatrix} 1 & -2 & -6 \\ -2 & 2 & -5 \\ 2 & 1 & 8 \end{pmatrix}$. Is the matrix A diagonalizable? (3+5)

3. Define a linear transformation $T : P_2 \to P_2$ be $T(ax^2+bx+c) = 2ax^2 + (2a+2c)x - 2a + 2b$.

   a) Find $[T]_\alpha^\alpha$ where α is the standard basis of P_2 consisting of x^2, x and 1.

   b) Deduce the change of basis matrix from α to the basis β for P_2 given by $\{x^2+x, x^2-1, x-1\}$.

   c) Find the change of basis matrix for β to α.

   d) Hence find $[T]_\beta^\beta$. (2+2+2+2)

4. a) Prove that if y_1 is a known solution of $y'' + p(x)y' + q(x)y = 0$, then a second linearly independent solution is

   $$y_2 = y_1 \int \frac{e^{-\int p\,dx}}{y_1^2}\,dx$$

   b) Obtain the general solution of the ODE, $xy'' - y' = x$, using the method of variation of parameters. (4+4)

Group – B

(*Answer Question No. 1 which is compulsory and **any two** of the rest*)

5. a) Show that the Cauchy-Riemann conditions are equivalent to the single condition $\frac{\partial f}{\partial z^*} = 0$. 2

   b) Use the generating function of the Hermite polynomials, $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)t^n}{n!}$, to prove that $H'_n(x) = 2nH_{n-1}(x)$. 2

Please Turn Over

6. a) Find the Laurent series expansion of $f(z)=\frac{1}{e^z-1}$ valid in the region $0<|z|<2\pi$.

b) Using the integral representation of the Bessel's function,

$J_n(z)=\frac{1}{2\pi i}\oint_C u^{-n-1}\exp\left(\frac{z}{2}\left(u-\frac{1}{u}\right)\right)du$, where C is the unit circle $|u|=1$ show that

$J_n(z)=\frac{1}{\pi}\int_0^{\pi}\cos(n\theta-z\sin\theta)\,d\theta$. (4+4)

7. a) Find the Green's function for the following ordinary differential equation

$\frac{d^2y}{dx^2}=f(x), y(0)=0 \ \ and \ \ y'(a)=0, 0\le x\le a$

b) For the second-order linear differential equation

$\frac{d^2w}{dz^2}+\left(\frac{A_1}{z}+\frac{B_1}{z-1}\right)\frac{dw}{dz}+\left(\frac{A_2}{z^2}+\frac{B_2}{(z-1)^2}-\frac{C_2}{z(z-1)}\right)w=0$ prove that the sum of its characteristic exponents is equal to unity. (4+4)

8. a) Evaluate $\int_0^{\infty}\frac{dx}{x^4+1}$ using contour integration.

b) Prove that the Laguerre polynomials $L_n(x)$ are orthogonal in $(0, \infty)$ with respect to the weight function e^{-x}. (6+2)

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M. Sc. (1^{ST} Year) 1^{ST} SEMESTER EXAMINATION 2019

Subject : Physics

Paper : Phy/Th/1S/102

Classical Mechanics

Time : 2 Hours **Full Marks : 40**

Group – A

(Question **No. 1** is compulsory and answer **any two** from the rest)

1. a) Show that for any function f(q, p, t) and Hamiltonian H

$$\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}$$

State the condition under which the function f is a constant of motion.

b) Show that if $\dot{p} = -\alpha\dot{q}\ (\alpha > 0)$, $\dot{q} = \frac{p}{m}$ then areas in the (q, p) plane decrease exponentially with time. 2+2=4

Answer any two of the following questions. **2 x 8=16**

2. a) A bead of mass m slides without friction in a uniform gravitational field on a vertical circular hoop of radius R. The hoop is constrained to rotate at a fixed angular velocity Ω about its vertical diameter. Assuming θ to be the position of the bead on the hoop measured from the lowest point show that there exists a first integral of motion for the system which is not equal to the mechanical energy.

b) For what values of the parameters α and β is the following transformation canonical?

$$Q = q^{\alpha}\cos\beta p, \quad P = q^{\alpha}\sin\beta p$$ 5+3

3. a) Derive Hamiltons canonical equations of motion from the variational principle.

b) A particle of charge q moves in a static electromagnetic field with potentials $V = -a\ln\rho$ and **A** = (- By/2, Bx/2, 0), where $\rho = \sqrt{x^2 + y^2}$ and a and B are constants. Deduce the Hamiltonian and write the Hamiltons canonical equations of motion. 3+5=8

4. a) A bead of mass m slides without friction on the curve $y = x^2$. The y-axis being vertically up. Find the equation of motion for x by using the method of Lagrange multipliers. Calculate the reaction of the constraint.

b) Write the Hamilton-Jacobi equation for the motion of a freely falling particle under gravity. Obtain the solution for Hamilton's characteristic function. (2+2)+4=8

Please Turn Over

Group – B

(Question **No. 5** is compulsory and answer **any two** from the rest)

5. a) Show that the sum of any two orthogonal space like vectors is space like.

   b) Show that inertia tensor is a symmetric tensor of rank two. 2+2=4

6. a) A rigid body, fixed in three dimensions, undergoes three successive rotations by angles $(\frac{\pi}{2}, \pi, \pi/2)$, respectively, along (z, x', z') - symmetry axes. Obtain the transformation matrix A resulting from three rotations using matrix operations. Find the symmetry axis and the angle of rotation, if any, under $\vec{X}' = A\vec{X}$.

   b) The inertia tensor of a rigid body having mass m fixed at one point, is given by $I_{ij} = m\begin{bmatrix} 2 & -3 & 0 \\ -3 & 2 & -3 \\ 0 & -3 & 4 \end{bmatrix}$ units. It is free to rotate about an arbitrary axis of rotation given by $\vec{\omega} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find the angular momentum and kinetic energy of the rigid body.

   (2+2)+(2+2)=8

7. a) Show that $c^2 d\gamma = \gamma^3 v dv$, where $\gamma = 1\Big/\sqrt{1-\frac{v^2}{c^2}}$.

   b) A relativistic particle having rest mass m collides with another particle at rest having rest mass M and gets stuck to it and the combined mass starts moving. Show that the rest mass of the combined mass increases.

   c) Two frames, O and O' are in relative motion along x-axis. O' is moving with speed c/2, where c is velocity of light. In frame O, two separate events occur at (x_1, t_1) and (x_2, t_2). In frame O', these events occur simultaneously. Show that $\frac{x_2 - x_1}{t_2 - t_1} = 2c$. 2+3+3=8

8. a) A symmetrical top is freely rotating with angular velocity $\vec{\omega}$. The PMI of the body are denoted by I, I and I_3 respectively about three principal body axes.

   i) Write down the Euler's equations of motion of the freely rotating symmetrical top.

   ii) Show that the angular velocity vector $\vec{\omega}$ precesses in a cone about the body symmetry axis with time period, $T_p = \frac{(I_3 - I)\omega_3}{2\pi I}$, where ω_3 is the component of angular velocity about the body symmetry axis.

   b) On her 21st birthday, an astronaut takes off in a spaceship moving at a speed $\frac{12}{13}c$. After 5 years have elapsed on her digital watch, she turns around and heads back at the same speed to rejoin her twin brother staying at home. How old is each twin at their reunion?

   (2+3)+3=8

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M. Sc. (1ST Year) 1ST SEMESTER EXAMINATION 2019

Subject : Physics

Paper : Phy/Th/1S/103

Quantum Mechanics - I

Time : 2 Hours **Full Marks : 40**

The figures at the end of question indicate full marks.

(*Use **separate Answer Script** for each group*)

Group – A

(Answer Question no. 1 and any two from the rest)

1. a) Evaluate the commutator $[L_z, p^2]$.

   b) Show that $-i\hbar\frac{d}{dx}$ is an Hermitian operator. 2+2=4

2. a) Solve the problem of the one dimensional rectangular potential barrier of height 'V_0' and width 'a' when a particle of mass m having energy $E < V_0$ is incident upon the barrier from the left. Hence show that the transmission coefficient is given by

$$T = \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)}\sinh^2 \beta a} \quad \text{where } \beta^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

   b) An electron has a speed of 500 m/s with an accuracy of 0.004%. Calculate the certainty with which we can locate the position of the electron.

   c) Prove that

$[xp_x, H] = \frac{i\hbar}{m}p_x^2 + x[p_x, V]$ where $H = \frac{p_x^2}{2m} + V$, and V is the potential energy operator.

4+2+2=8

3. a) Obtain the matrix of Clebsh-Gordan coefficient for $j_1 = 1/2$ and $j_2 = 1/2$

   b) The vector J gives the sum of angular momentum J_1 and J_2. Prove that $[J_x, J_y] = i\hbar J_z$. Is $J_1 - J_2$ an angular momentum operator? 4+(2+2)=8

4. a) Find the expectation value of the potential energy in the n^{th} state of the harmonic oscillator.

   b) For an eigenstate of J^2 and J_z, show that

$$\langle J_x \rangle = \langle J_y \rangle = 0$$

$$\langle J_x^2 \rangle = \langle J_y^2 \rangle = \frac{1}{2}\hbar^2[j(j+1) - m^2]$$

3+(2+3)=8

Please Turn Over

Group – B

(Answer Question no. 5 and any two from the rest)

5. a) A particle is trapped in an infinite square well of width "a" at its bottom and its unperturbed wave function is given $\psi_n^{(o)} = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$. If the system is perturbed by raising the floor by a constant potential V_0, find the 1st order correction to the energy in its nth state.

   b) Write down the condition of validity of WKB approximation in quantum mechanics and comment of the mass and energy of the particle to fulfill the validity. 2+2=4

6. a) A particle of mass m moves in a cubical box of sides 'L' with potential zero inside and infinite outside the box. Find the ground state energy eigen value.

   b) What are the basic features of Interaction representation in quantum mechanics? Show that this representation satisfies a similar time-dependent Schrodinger equation with H (unperterbed Hamiltonian) replaced by perturbed Hamiltonian H'. 4+(1+3)=8

7. a) Calculate the 1st order correction to the ground state energy of a 1D anharmonic oscillator of mass m and angular frequency ω subject to a potential $V(x) = \frac{1}{2} m\omega^2 x^2 + bx^4$. Given the ground state wave function as

$$\psi_n^{(o)} = (m\omega/\pi\hbar)^{1/4} \exp(-m\omega x^2/2\hbar)$$

   b) Treating spin-orbit interaction $H' = W(r)\, \vec{L}.\vec{S}$ as perturbation, find the splitting of Na-atom to explain D-lines. 3+5=8

8. a) Obtain an expression of WKB wave function for a system with E > V.

   b) Obtain an expression of the frequency of the spectral lines in normal Zeeman effect. 5+3=8

Time