

DHWU

M.Sc. (1st Year) 2nd Semester Examination, 2018

Subject: Physics

Paper: Phy/Th/2S/201/18

Quantum Mechanics II

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks.

Answer any two questions from each group

(Use separate answer scripts for each group)

Group-A

1. a) The operators S_x and S_y are defined as : $S_x = \frac{1}{2}\hbar\{|+\rangle\langle -| + |-\rangle\langle +|\}$ and $S_y = \frac{1}{2}\hbar\{-i|+\rangle\langle -| + i|-\rangle\langle +|\}$. Obtain the matrix representation of S_{\pm} .
- b) Find the outer product $|\alpha\rangle\langle\beta|$ where $\langle\alpha| = \{+3i, 2-i, 4\}$ and $\langle\beta| = \{2, +i, 2+3i\}$.
- c) Construct the spherical tensor of rank 2 out of two different vectors \vec{U} and \vec{V} . Express $T_{\pm 2}^{(2)}$ and $T_0^{(2)}$ in terms of $U_{x,y,z}$ and $V_{x,y,z}$. 3+3+4=10

2. a) Two spin 1 objects interact with each other with an external vector field \vec{A} in such a way that the Hamiltonian for the system can be written as $H = \frac{A}{\hbar^2}(\vec{S}_1 \cdot \vec{S}_2) + \frac{\vec{A} \cdot (\vec{S}_1 + \vec{S}_2)}{\hbar}$, where $\vec{A} = |A|$ is the magnitude of the field \vec{A} . Find the energies and degeneracies of the ground state and 1st excited state.
- b) Write down Dirac equation in momentum space for particle. Hence, show that $\bar{u}^{(s)}u^{(s)} = 2m$. Notations have their usual meaning.
- c) Classify the following operators as *polar vectors*, *pseudovectors*, *ordinary scalars* and *pseudoscalars* : a) $\vec{L} \cdot \vec{S}$; b) $\vec{L} \cdot (\vec{x} \times \vec{S})$ 5+3+2=10

3. a) Show that orbital angular momentum is not conserved for free Dirac particle. How is the concept of spin introduced in this regard?
- b) Dirac spinor for particle is given by :

$$u^{(s)} = N \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi^{(s)} \end{pmatrix}, \text{ where } N = \sqrt{E+m};$$

Show that $\sum_{s=1,2} u^{(s)}(\vec{p}) \bar{u}^{(s)}(\vec{p}) = \not{p} + m$ (Notations have their usual meaning.) (3+3)+4=10

4. a) Starting from the expression of the infinitesimal rotation operator given by:

$$\mathcal{D}(\hat{\mathbf{n}}, d\varphi) = 1 - i \frac{\vec{\mathbf{J}} \cdot \hat{\mathbf{n}}}{\hbar} d\varphi$$

establish the fundamental commutation relations of angular momentum, where $\mathcal{D}(\hat{\mathbf{n}}, d\varphi)$ is the rotation operator about an axis characterized by unit vector $\hat{\mathbf{n}}$ by an infinitesimal angle $d\varphi$ and $\vec{\mathbf{J}}$ is the angular momentum operator.

- b) Discuss the Feynman-Stueckelberg interpretation of negative energy solutions.

- c) Obtain the Klein-Gordon equation from Dirac equation. 4+3+3+10

Group-B

5. (a) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, for a harmonic perturbation: $H'(\mathbf{r}, t) = 2 H'(\mathbf{r}) \cos(\omega t)$, obtain transition probability from a discrete state i to a discrete state k for stimulated emission and absorption.

- (b) An atom is placed in an electromagnetic field. Using dipole approximation, obtain an expression of transition probability induced by the radiation.

Why does the dipole transition must be accompanied by a parity change between the states?

$$4+(5+1)=10$$

6. (a) Assuming the asymptotic solution of the Schrodinger equation for scattering of a particle by a spherically symmetric potential $V(r)$ being of the form (terms having their usual meanings):

$$\Psi(\mathbf{r}) = e^{ikz} + f(\theta, \phi) \frac{e^{ikr}}{r},$$

obtain an expression of total scattering cross section using partial wave decomposition method. Obtain optical theorem from it.

Given $\Psi_{in}(\mathbf{r}) = e^{ikz} = \sum_{l=0}^{\infty} A_l j_l(kr) P_l(\cos\theta)$ with $A_l = (2l+1)i^l$.

- (b) How will the expression of scattering cross section be modified for indistinguishable particles?

- (c) Write down all possible wave functions (spin-orbitals) of the ground and 1st excited state of He atom. 5+2+3=10

7. (a) Find the total scattering cross section of a particle, in the low energy limit, by a square well potential of the form

$$V(r) = -V_0, \text{ for } 0 < r < a$$

$$= 0, \text{ for } r \geq a, \text{ } a = \text{width of the potential well.}$$

What is the maximum value of the above total scattering cross section?

- (b) What is first Born approximation in connection with the scattering problem?

Using standard expression of scattering amplitude $f(\theta, \phi)$ due to 1st Born approximation, find an expression of the differential scattering cross section for a shielded Coulomb potential of the form: $V(r) = \frac{Z_1 Z_2 e^2 e^{-r/l}}{r}$, l being the shielding distance. Show that under certain condition it reduces to Rutherford scattering formula.

$$(4+1)+(3+2)=10$$

8. (b) Starting from standard expression of 1st order transition amplitude $C_k^{(1)}(t)$, obtain Fermi Golden rule in connection with transition due to time dependent perturbation.

(b) Obtain selection rule for electric dipole transition of a linear harmonic oscillator.

(c) Assuming the asymptotic solution of the Schrodinger equation for scattering of particle by a spherically symmetric potential of the form (terms having their usual meanings):

$$\Psi(\mathbf{r}) = e^{ikz} + f(\theta, \phi) \frac{e^{ikr}}{r},$$

find the relation of differential scattering cross section with scattering amplitude.

- (d) Write down the wave functions in terms of space and spin orbitals ¹S-state of two-electron atom.

$$3+3+3+1=10$$

Moderated By :
DHWU

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Signature of Paper-Setter in Full
and Date

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1.

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DHWU

M.Sc. 2ND SEMESTER EXAMINATION 2018

Subject : Physics

Paper : Phy/Th/2S/202

Statistical Mechanics

Time : 2 Hours

Full Marks : 40

The figures in the margin indicate full marks.

Answer two questions from each group

(Use separate answer script for each group)

Group – A

1. a) Show that for a system in a canonical ensemble $\langle (\Delta E)^2 \rangle = k_B T^2 C_V$ where C_V is the specific heat at constant volume.
- b) A solid containing non-interacting paramagnetic atoms, each have a magnetic moment equal to 1 Bohr Magnetron (BM), is placed in a magnetic field of 3 Tesla. Assuming that the atoms are in thermal equilibrium with the surrounding. Find the temperature at which the solid must be cooled so that more than 60% of atoms are polarised with magnetic moment parallel to magnetic field. (1BM = 9.27×10^{-24} J/T)
- c) If a system consisting of two subsystems is in mechanical equilibrium and the volume V_1 and V_2 of two sub-systems are in such a way that the entropy S is maximum, then show that statistical pressure may be defined by

$$\frac{\pi}{\tau} = \left(\frac{\partial S}{\partial V} \right)_{U, N} \text{ where terms have their usual meaning.}$$

- d) Under what conditions the postulates of equal a-priori probability are applicable?

$$3+3+3+1 = 10$$

2. a) What is Gibb's paradox? How it is resolved?
- b) Derive Sackur-Tetrode equation for entropy of an ideal gas.
- c) Show that the partition function Z_N for an extreme relativistic gas consisting of N -non-interacting molecules with energy-momentum relationship $\epsilon = cp$ is given by

$$Z_N = \frac{1}{N!} \left[8 \pi V \left(\frac{kT}{hc} \right)^3 \right]^N. \text{ Hence find an expression for the pressure of the gas.}$$

$$(1+2)+3+(3+1) = 10$$

3. a) For a two dimensional system, show that the density of states is given by

$$D(\epsilon) = \frac{gm}{2\pi\hbar^2} \text{ for non-relativistic energy.}$$

$$= \frac{gE}{2\pi(\hbar c)^2} \text{ for relativistic energy, where the symbol have their usual meaning.}$$

$$= \dots$$

Please Turn Over

- b) Establish the relation $C_v = k\beta^2 \left(\frac{\partial}{\partial \beta^2} (\beta F) \right)_V$ where F is the free energy and the symbols have their usual meaning.
- c) A system consisting of N spatially separated sub-systems, is in thermal equilibrium with a heat reservoir at absolute temperature $T = \frac{\epsilon}{k}$, where ϵ is the energy. If each sub-system has non-degenerate energy levels of energy 0, ϵ , 2ϵ , 3ϵ find
- the partition function,
 - the mean energy, and
 - the entropy of the system.
- (2+2)+3+3 = 10

Group - B

4. a) Defining the density matrix ($\hat{\rho}$) for a pure quantum statistical state, establish the Liouville's theorem.
- b) What are the basic differences between classical and quantum statistical mechanics?
- c) Can you interpret the following matrix,

$$\hat{\rho} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

as a density matrix? Justify your answer.

(2+5)+2+1 = 10

5. a) What do you mean by the Bose-Einstein (BE) condensation temperature (T_0) for an ideal Bose gas? Why there is a sharp decay in the amplitude of the specific heat (C_v) below T_0 ? Why the Bose-Einstein condensation sometimes called as "condensation in momentum space"?

- b) Show that the entropy of an ideal Bose gas for $T < T_0$ is

$$S = \frac{2}{3} (C_v)_{T < T_0}$$

Given that the energy of the ideal Bose gas for $T < T_0$ is $E_{T < T_0} = \frac{3}{2} \frac{F_{\frac{5}{2}}^{(0)}}{F_{\frac{3}{2}}^{(0)}} nkT \left(\frac{T}{T_0} \right)^{\frac{3}{2}}$

where the symbols carry their usual meaning.

(2+2+2)+4 = 10

6. a) What do you mean by weak and strong degeneracies for an ideal *Fermi-Dirac* gas?
- b) Establish a relation between the energy (E) and the volume (V) for an ideal *Fermi* gas and show that the pressure can be written as

$$P = \frac{2E}{3V}$$

- c) Compare the energies of the ideal Bose-Einstein and ideal *Fermi-Dirac* gas while the temperatures of both the gases are tending towards absolute zero. Discuss about the energy degeneracies for both the gases at that temperature.

(1+1)+(2+2)+(2+2) = 10

M.Sc. 2nd

Time : 2 Hours

1. a) Draw sketch (UJT). Exp
b) The circuit The switch Laplace

2. a)

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M.Sc. 2ND SEMESTER EXAMINATION 2018

Subject : Physics

Paper : Phy/Th/2S/203

General Electronics

Time : 2 Hours

Full Marks : 40

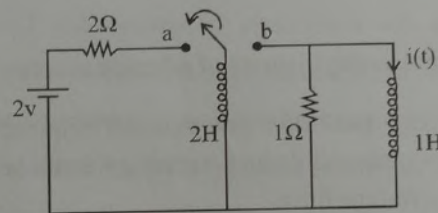
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(Use separate answer script for each group)

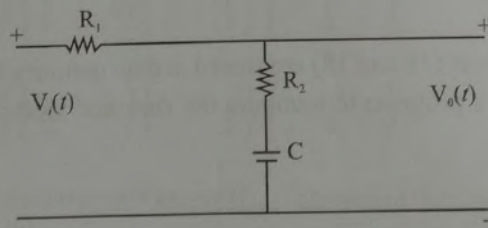
Group – A

1. a) Draw sketches to show the construction and equivalent circuit of a unijunction transistor (UJT). Explain its principle of action using the characteristic curves.
- b) The circuit in the fig. below was initially in steady state with switch 'S' is position 'a' at $t = 0$. The switch then goes from 'a' to 'b' position. Find $i(t)$ for the following network using Laplace transforms at $t > 0$.



(1+1+4)+4 = 10

2. a) Define transfer function in connection with Laplace transformation. Find the transfer function for the following network.



- b) Is it possible to decrease the non linear distortion in a feedback amplifier using a negative feedback? Explain.
- c) Determine the highest angular frequency allowed in a demodulation circuit in terms of capacitance and resistance of the circuit with the angular frequency of input signal.
- d) The peak to peak value of an AM voltage has a maximum value of 8V and a minimum value of 2V. What are the percentage modulation and the amplitude of the unmodulated carrier?

(1+3)+2+2+2 = 10

Please Turn Over

3. a) Draw the circuit diagram of a two stage RC coupled amplifier. Calculate the low frequency voltage gain of a RC coupled one stage amplifier. Finally show that at low frequency the value of phase angle is greater than 180° .

b) A single tuned amplifier has the following parameters :

$$L = 120 \mu\text{H}$$

$$h_{oc} = 50 \times 10^{-6} \Omega^{-1}$$

$$C = 100 \mu\text{H}$$

$$h_{ic} = 100$$

$$R = 10 \Omega$$

$$h_{ic} = 2.5 \text{ k}\Omega \text{ and load resistance } R_T = 10 \text{ k}\Omega$$

Calculate (i) the resonant frequency, (ii) the bandwidth and (iii) the maximum voltage gain.
(1+3+2)+(1+2+1)=10

Group – B

4. a) Draw the circuit diagram of an integrator using op-amp. Obtain the output voltage form when 1 V dc is applied to its input with $R = 500 \text{ k}\Omega$, $C = 1 \mu\text{F}$ and supply voltage $= \pm 12 \text{ V}$.

b) Draw the circuit diagram and explain the working principle of a weighted digital-to-analog converter. What is the disadvantage of this D/A converter and how can we overcome this?

$$(1+3)+(4+1+1)=10$$

5. a) Draw the circuit diagram and timing diagram of a decade counter. What is RAM?

b) Design a first order active high-pass filter with a cut-off frequency of 1 kHz and a pass band gain of 2. (Give circuit diagram and deduce necessary formula to design). Give the basic circuit diagram of this passive-type filter.

$$(3+1)+(5+1)=10$$

6. a) Draw the circuit diagram of an astable multivibrator and obtain an expression of its frequency. To obtain a square wave of frequency $f = 1 \text{ kHz}$, give the values of the necessary circuit components?

b) Two digit of BCD numbers (16 and 18) are stored at data memory location X and X + 1 of a micro-processor. Write a program to compute the sum and store the decimal result at the location X + 2.

$$(5+2)+3=10$$

Time : 2 Hours

1. a)

2.