M.Sc. (1st Year) 1st Semester Examination, 2018

Subject : Physics

1st Paper : Phy /Th / 1S / 101/ 18

Mathematical Methods

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks.
Answer Two questions from each group
*(Use **separate** Answer script for each group)?*

Group - A

1. a) Let's say φ_1 and φ_2 be the two ordered pair of vectors in a complex plane. Then establish the following inequality,

$$(\varphi_1, \varphi_1)(\varphi_2, \varphi_2) \geq |(\varphi_1, \varphi_2)^2|;$$

Where (φ_1, φ_2) is the scalar product of φ_1 and φ_2

b) Determine C_1, C_2 and C_3 such that the following linear equation is satisfied.

$$(1,-1,4) = C_1(2, 3, 5) + C_2(-1, 0, 6) + C_3(1, 3, 11),$$

What conclusions can you draw from your result ?

c) Evaluate the following integral :

$$\oint_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$$

where C is the circle $|z| = 1$. 4+2+4=10

2. a) State the converse of Cauchy's theorem.

b) Evaluate the following integral :

$$\frac{1}{2\pi i} \oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where C is the circle $|z| = 3$

c) Find the inverse of the matrix,

$$A = \begin{pmatrix} \cos\varnothing & \sin\varnothing \\ -\sin\varnothing & \cos\varnothing \end{pmatrix}$$

d) Check, whether $\{(i, 1+i)(2, 1-i)\}$, forms a *basis* in C^2 or not. 2+4+2+2=10,

3. a) Check, whether $u = e^{-x}$ (x sin y − y cos y) is harmonic or not

b) Evaluate the following integral : $\int_0^\infty \frac{1}{x^6+1} dx.$

c) Can the set of all real solutions of the differential equation $d^2y/dx^2 + \omega^2 y = 0$, ($\omega$ is constant) form a linear vector space over a real field ? 4+5+1=10

Please Turn Over

Group - B

4. a) Find the type of singularity arises from the given differential equation,

$$8x^2\frac{d^2y}{dx^2}+10x\frac{dy}{dx}+(x-1)y=0$$

b) If y_1, y_2 are the solutions of the linear homogeneous differential equation,

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)\,y=0$$

show that the one of the solutins is linearly dependent on the other.

c) Prove that, the solution of an inhomogeneous 2nd order differential equation can be written as $y(x)=\int_a^b G(x,z)f(z)dz$, where $G(x,z)$ is the Green function of the differential equation.

d) Solve the inhomogeneous differential equation using Green function method.

$$\frac{d^2y}{dx^2}=x^2,\text{ where } y(0)=0=y'(1)$$

2+3+3+2=10

5. a) Prove the orthogonality relation of Bessel's function

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx=0,\ \alpha\neq\beta$$

$$=\frac{1}{2}[J_{n+1}(\alpha)]^2,\ \alpha=\beta$$

where α and β are the roots of $J_n(x)=0$

b) State and prove the convolution theorem of Laplace's transformation.

c) If $f(x)=0 \quad -1<x\leq 0$

$= x \quad 0<x<1,$

then show that, $f(x)=\frac{1}{4}P_0(x)+\frac{1}{2}P_1(x)+\frac{5}{16}P_2(x)-\frac{3}{32}P_4(x)+\cdots$

4+3+3=10

6. a) Prove the recurrence relations :-

i) $\frac{d}{dx}[x^{-n}J_n(x)]=-x^{-n}J_{n+1}(x)$

ii) $P_n'(x)=xP_{n-1}'(x)+nP_{n-1}'(x)$

b) In an electrical circuit with e.m.f.E(t), resistance R and inductance L, the current I builds up at the rate given by $L\frac{di}{dt}+Ri=E(t)$. If the circuit is switched on at $t=0$ and disconnected at $t=a$, find the current i at any instant, using Laplace transform.

(3+3)+4=10

M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2018

Subject : Physics

2^{nd} Paper : Phy /Th / 1S / 102/ 18

Classical Mechanics

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks.

Answer Two questions from each group

*(Use **separate** Answer script for each group)*

Group - A

1. a) Two identical parallel pendulums are coupled by a massless spring at their bobs. Considering small amplitude oscillations, obtain normal frequencies. Write down the equations of motion for symmetric and antisymmetric modes of oscillation.

   b) State Hamilton's variational principle.

   Consider a curve passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$. When it revolves around a vertical axis, it traces a surface of revolution. Find the equation of the curve for which the surface area is a minimum. (4+2)+(1+3)=10

2. a) Show that following equations generate a cononical transformation and also find the corresponding generating function.

$$P = 2\left(1 + q^{\frac{1}{2}} \cos p\right) q^{\frac{1}{2}} \sin p, \quad Q = ln(1 + q^{\frac{1}{2}} \cos p) .$$

   b) Obtain the solution of a harmonic oscillator described by the Hamiltonian

   $H = \frac{1}{2}p^2 + \frac{1}{2}\omega q^2$ and generating function given by $F = \frac{1}{2}\omega q^2 \cot(2\pi Q)$.

   c) If the Hamiltonian involves time explicitly what other method do you know to seek a canonical transformation and mention the basic features of that method. (3+1)+4+2=10

3. a) For infinitesimal contact transformation, if it corresponds to space translation, prove that linear momentum of the system is the generator and it is a constant of motion.

   b) Prove the Jacobi-Poisson's theorem in case of functions having explicit time dependence.

   c) Using Hamilton Jacobi method, solve the equation of motion of a ID harmonic oscillator. 4+3+3=10

Group - B

4. a) The effective potential of a heavy symmetrical top under gravity is given by :

$V(\theta)=\frac{1}{2}I\left(\frac{b-a\cos\theta}{\sin\theta}\right)^2 Mgl\cos\theta$, where $I_1 a = I_1\dot{\phi}\sin^2\theta + I_1 b\cos\theta$ and $I_1 b = I_3(\dot{\phi}\sin\theta+\dot{\psi})$.

Notations have usual meaning. Obtain the condition for steady precession of the top.

b) The components of a covariant vector, A_μ and B_μ are (-1,1,0,2) and (0,-1,2,-1) respectively. Find A^μ. Hence, evaluate A . B.

c) Show that $\gamma v_, = \gamma^3 dv$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ jv is the velocity along x-direction. 5+3+2=10

5. a) Two particles move in an inertial frame. S. B has speed $\frac{c}{3}$ w.r.t A and A has speed $\frac{c}{3}$ w.r.t.S. What is the speed of B w.r.t. S?

b) Show that a vector orthogonal to a time like vector must be space like.

c) Show that the lagrangian of a relativistic particle of rest mass m_0 is

$L = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} - V.$ Where the notations have their usual meaning. Hence obtain the equations of motion of the particle. 2+3+5=10

6. a) π measons coming out of an accelerator have a velocity of 0.99c. If they have a mean life time of 2.6×10^{-8} S in the rest frame, how far can they travel before decay ?

b) A pion (π^+) at rest decays into a muon (μ^+) and a muon type anti-neutrino ($\bar{\nu}_\mu$). Assuming neutrinos, to be massless, find the energy of the outgoing muon in terms of rest masses of the given particles.

c) Define Euler angles. Hence obtain an expression for angular velocity ω of a rigid body in its body-fixed frame in terms of Euler angles. 3+3+4=10

M.Sc. (1^{st} Year) 1^{st} Semester Examination, 2018

Subject : Physics

3^{rd} Paper : Phy /Th / 1S / 103/ 18

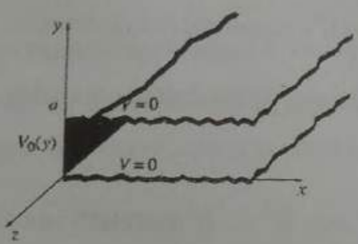
Electrodynamics

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks.
Answer Two questions from each group
(Use ***separate*** *Answer script for each group)*

Group - A

1. a) Two infinite grounded metal plates lie parallel to xz plane, one at y=0, the other at y=a as shown in the figure. The left end, at x=0, is closed off with an infinite strip insulated from the plates and maintained at a specific potential $V_0(y)$. Find the potential inside the slot.



b) The potential $V_0(\theta)=k\sin^2\frac{\theta}{2}$ is specified on the surface of a hollow sphere of radius R. Find the potential inside and outside the sphere.

c) An electric dipole of moment $\vec{p}$ is placed with axis vertical at a height h above a large horizontal grounded conducting plane. Calculate the force of attraction on the dipole exerted by the conducting plane. 4+3+3=10

2. a) A plane electromagnetic wave is incident normally on a conductor of electrical conductivity σ. Show that electromagnetic wave is damped inside the conductor and finds it skin depth.

b) The electric field of a plane wave in sea water at its surface (z=0) is given by
$\vec{E}=\hat{i}\,100\cos(10^7\pi t)$ V/m

For sea water, $\varepsilon=72\varepsilon_0$ F/m, $\mu=4\pi\times10^{-7}$ H/m and $\sigma=4$ mho/m.

i) Determine whether the sea water acts as a good conductor or not.

ii) Find the phase velocity and the skip depth.

Please Turn Over

iii) At what depth below the surface does the amplitude of $\vec{E}$ drop to 1% of its value at the surface ?

c) Explain the concept of displacement current and show its importance. 4+(1+2+1)+2=10

3. a) What is waveguide ? Show considering TM waves, that there is cut off frequency for propagation between parallel conducting plates. Why TEM waves cannot propagate in rectangular waveguides ?

b) The electric field E of a plane wave in air is given by

$\vec{E}=[\hat{i}4\times10^{-6}\cos(10^{7}\pi t-kz)+\hat{j}4\times10^{-6}\sin(10^{-7}\pi t\text{-}kz)]$V/m

Find the value of k, the magnetic field and the Poynting vecor. (1+5+1)+3=10

Group - B

4. a) Write down Maxwell's equations in convariant form.

b) Obtain the differential form of Gauss's law and Faraday's law from the covariant form of Maxwell's equations.

c) Starting from Maxwell's equations in four dimensions, porve that $\frac{\partial J_\mu}{\partial x_\mu}=0$

d) Assuming the Lorentz gauge condition, prove that $\frac{\partial^2 A_\mu}{\partial_\nu \partial x^\nu}=-\mu_0 J_\mu$, symbols have their usual meaning. 2+4+2+2=10

5. a) Show that $\vec{E}\ .\ \vec{B}$ and $E^2-c^2B^2$ are relativistically invariant.

b) The angular and frequency dependence of radiation energy of a moving charged particle is given by :

$$\frac{dI(\omega)}{d\Omega}=\frac{q^2\omega^2}{32\pi^3\varepsilon_0 c^2}\left|\int_{-\propto}^{\propto} e^{i\omega(t-\frac{\hat{n}\cdot\vec{r}}{c})}\left[\hat{n}\times(\hat{n}\times\vec{\beta})\right]dt\right|^2$$

where symbols have their usual meaning.

Explain Cherenkov radiation starting from the above equation. What is Cherenkov cone ? (2+2(4+2)=10

6. a) An insulting circular ring of radius b lies on the xy plane centered at the origin. It carries a linear charge density $\lambda=\lambda_0\cos\varphi$,where λ_0 is constant and φ is the usual azimuthal angle. The ring is now set spinning at a constant angular speed ω about z-axis. Calculate the power radiated.

b) Suppose in the rest frame we have $E_y=\alpha$, $B_y=\beta$, now we go to a frame moving with velocity $\vec{v}=v\hat{j}$.

I) Write down Lorentz transformation matrix Λ for this case.

ii) Find all the non-zero components of $\vec{E}$ and $\vec{B}$ in the new frame.

The symbols have their usual meaning. 5+5=10

M.Sc. (1st Year) 1st Semester Examination, 2018

Subject : Physics

4th Paper : Phy /Th / 1S / 104/ 18

Quantum Mechanics

Time : 2 Hours **Full Marks : 40**

The figures in the margin indicate full marks.
Answer Two questions from each group
*(Use **separate** Answer script for each group)*

Group - A

1. a) For a one dimensional harmonic oscillator, using creation and annihilation operators, show that $(\Delta x).(\Delta p_x) = (n+\frac{1}{2})\hbar$

   b) What is the zero point energy of harmonic oscillator ? How is it explained ?

   c) Operators J_+ and J_- are defined by $J_+ = J_x + iJ_y$ and $J_- = J_x - iJ_y$, where J_x and J_y are the x and y components of the general angular momentum **J**. Prove that

   $$J_+|j,m\rangle = [j(j+1) - m(m+1)]^{\frac{1}{2}}\hbar|j,m+1\rangle$$

   d) Show that commutating operators have common set of eigen functions.

   3+(1+1)+3+2=10

2. a) The rotational part of the Hamiltonian of a diatomic molecule is $\frac{1}{2I}(L_x^2 + L_y^2) + \frac{1}{I}L_z^2$,

   I being the moment of inertia of the diatomic molecule. Find the energy eigen values.

   b) Derive matrix for the operators J^2, J_z, J_x and J_y for $j=1$.

   c) Obtain the matrix of Clebsch-Gordan coefficient for $j_1=1/2$ and $j_2=1/2$.

   2+4+4=10

3. a) Show that Pauli spin matrices satisfy the following relation

   $\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$, where terms have their usual meaning.

   b) Prove that

   $[XP_x, H] = \frac{i\hbar}{m} P_x^2 + X[P_x, V]$, where $H = \frac{P_x^2}{2m} + V$, and V is the potential energy operator.

   c) Show that $-i\hbar\frac{d}{dx}$ is an Hermitian operator.

   d) Can we measure the kinetic and potpential energies of a particle simultaneously with arbitary precision ?

   3+3+2+2=10

Please Turn Over

Group - B

4. a) Prove that in Schrodinger representation of quantum mechanics any operator A satisfies the following relation :

$$\frac{d<A\ >}{dt} = \frac{1}{i\hbar}[A\ \ ,H] + \frac{\partial A}{\partial t}$$

What are the basic features of the interaction picture in quantum mechanics?

b) The potential of a 3D square-well potential of width 'a' is represented by

$V(r) = -V_o$ for $0<r<a$
$\quad\ = 0$ for $r>a$,

Find the acceptable wave functions in the regions $r<a$ and $r>a$.

c) Obtain the condition for validity of WKB approximation used in quantum mechanics ?

(3+1)+3+3=10

5. a) Calculate the 1^{st} order correction to the ground state energy of a ID anharmonic oscillator of mass m and angular frequency ω subject to a potential $V(x)=\frac{1}{2}m\omega^2x^2+bx^4$ Given the ground state wave function as

$$\psi_0^{(o)} = (m\omega/\pi\hbar)^4 exp(-m\omega x^2/2\hbar).$$

b) A rigid rotator having a moment of inertia I and electric dipole moment μ executes rotational motion in a plane. Estimate the 2nd order correction to energy when the rotator is acted upon by an electric field E in the plane of rotation. Given $E_n^0 = \frac{n^2\hbar^2}{2I}$.

c) Obtain an expression of WKB wave function for a particle with E>V 3+3+4=10

6. a) Treating relativistic interaction a $H' = -\frac{[E_n - V(r)]^2}{2m_0c^2}$ obtain an expression of its effect on the energy level of an one-electron system. The terms in H' have the usual meanings.

b) Using variational method, find the ground state energy of Helium atom.

Given $<1/r_{12}> = \frac{5}{4}Z_{eff}E_H$ and E_H=-13.6 eV.

c) Obtain energy eigen values of a ID harmonic oscillator using WKB approximation.

4+4+2=10